IOP Conference Series: Materials Science and Engineering

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IOP Conf. Series: Materials Science and Engineering **288** (2017) 012061 doi:10.1088/1757-899X/288/1/012061

Determine the Optimal Solution for Linear Programming with Interval Coefficients

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Abstract. The conventional linear programming model requires the parameters which are known as constants. In the real world, however, the parameters are seldom known exactly and have to be estimated. Linear programming with interval coefficients is one of the tools to tackle uncertainty in mathematical programming models. This paper presents a problem solving linear programming with interval coefficients. The problem will be solved by the algorithm general method to solve linear programming with interval coefficients. Thus, the best and the worst optimum solutions can be obtained, the upper bounds and lower bounds for the optimum value of the main problems can be determined.

1. Introduction

In the linear programming problem, sometimes coefficients in the model cannot be determined precisely so that is usually made in the form of estimation. One of the methods to resolve this issue is by using the interval approach, where the indeterminate coefficients transformed into the interval. This linear programming form is called Linear Programming with Interval Coefficient (LPIC). Interval coefficient indicates expansion the tolerance interval (or area) where the constant parameters are acceptable and meet the LPIC models.

In practice, a manager would like to know the range of optimum solutions that could be returned by the Linear Programming model with various settings of the uncertain coefficients. Unfortunately, the current state of the art provides little help. Classical sensitivity analysis allows a study of the effect on the solution of changes to single coefficient or very small groups of coefficients, but only to the extent that the optimal basis is not changed [1]. Wendell's' tolerance approach determines the maximum fractional change in any coefficient before the basis changes, and the 100% rule also considers simultaneous coefficient alterations that do not change the basis. There are no effective tools for examining the effects of many simultaneous changes to the coefficients that may change the basis. In any case, a hit-and-miss approach to the variation of the uncertain coefficients is unlikely to uncover the complete range of possible optimum objective function values [2].

At first, LPIC was not much discussed. Previous studies have focused on certain specific cases, for instance variable 0 - 1 or programming linier case with interval coefficient in the objective function only. LPIC topic was introduced widely in the year 1960-1980, starting from constrains models in the form of upper-bound and lower-bound. Although not related to the LPIC, these models have

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similarities that is the constraints model limited by the extreme point. Shaocheng transformed LPIC into two linear programming which has special characteristics [3]. One of the linear programs has the largest possibly feasible area and most favorable version of objective function to find the best possible optimum solution. Meanwhile the other linear programming had the smallest feasible area and both versions of the least favorable version of objective function to find the worst possible optimum solution. This Shaocheng method to overcome the problem LPIC with the terms: (a) restricted only nonnegative variable, and (b) only overcome inequality [4].

Inuiguchi and Sakawa deal with Linear Programming models involving interval coefficients in the objective function only. Their goal is to determine the closest single solution to all of the optimal solutions of the model under the uncertainties in the objective function. They use the minimax regret approach to find a solution that minimizes the largest difference in the values of any two versions of the objective function. From the point of view of solving the LPIC problem, this method has significant drawbacks: it deals only with interval objective functions, and it does not give the desired information about the range of the objective function values [5].

The extensions to equality constraints and to non-positive and unrestricted variables are important for practical reasons. Gass listed a number of uses for unrestricted variables in production smoothing applications, zero-sum two-person games, and numerical and statistical problems [6].

2. Methods

The research objective is the development of practical algorithmic tools for dealing with LPs in which the coefficients are known only approximately. The assumption is that any unknown coefficient can be expressed as an interval (a lower- and upper-bounded range of real numbers). This research develops methods that find the best optimum (highest maximum or lowest minimum as appropriate), and worst optimum (lowest maximum or highest minimum as appropriate), and the coefficient settings (within their intervals) which achieve these two extremes. The authors refer to the problem of finding the two extreme solutions and associated coefficient settings as Linear Programming with Interval Coefficients (LPIC).

3. Results and Discussion

3.1. *Linear Programming*

Linear programming is a mathematical tool which is developed to handle the optimization of a linear function subject to a set of linear constraints. The linear programming is very important in the area of applied mathematics and has a large number of uses and applications in many industries. Some current applications include; allocation of resources, transportation and scheduling operations [7].

The standard form of the linear programming model is as follows: [8]

$$\text{Maximization } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

subject to

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m \end{gathered}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \dots, x_n \ge 0$$

Function which maximizes $c_1x_1 + c_2x_2 + \cdots + c_nx_n$is called the objective function.

$x_1, x_2, \dots, x_n$is a decision variable.

c_j, b_i *and* a_{ij} (for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$)are the model parameters.

Definition 1 Feasible solution is a solution that all constraints are met. Non feasible solution is solution at least one constraints violated [8].

Definition 2 Feasible region is a collection of all feasible solutions [8].
Definition 3 The optimal solution is a feasible solution that has the most favorable value of the objective function [8].

3.2. Characteristics of Linear Equations with Coefficients Interval

Definition 4Let$[a_1, a_2]x_1 + [b_1, b_2]x_2 = (\geq, \leq)[c_1, c_2]$be constraint given in linear programming problem with interval coefficients. Shifting constraints is constraints parallel movement from one position to another without changing the slope. This shift is only caused by changes in the Right Hand Side from the value in $[c_1, c_2]$ to another value in $[c_1, c_2]$. [5]

Definition 5Let$[a_1, a_2]x_1 + [b_1, b_2]x_2 = (\geq, \leq)[c_1, c_2]$be certain constraints (or objective function) in a linear programming problem with interval coefficients. Slope is the change in slope of the given constraints (or objective function). Slope caused by changes in at least one of the coefficients interval associated with one variable from the value in the interval for other values in the interval [9].

Definition 6**.** Reversal of linear constraints (or objective function) of a linear programming problem is a special kind of slope that occurs when there is a change simultaneously from all signs of the coefficients on the LHS (that is, all interval coefficient associated with the variable range of zero) [9].
Interval Coefficient Effects in Linear Programming: Let$C$be the set of constraints with the interval coefficients, $C_1$and$C_{II}$are two sets of different constraints generated from$C$by using different extreme versions. Feasible area$S_I$andS_{II}generated by$C_I$and$C_{II}$are explained by one of these possibilities: [9]

1. $S_I \subseteq S_{II}$or$S_{II} \subseteq S_I$, is a feasible area entirely contained in another feasible area.
2. $S_I \neq S_{II}$and$S_I \cap S_{II} \neq \emptyset$,is a feasible partially offs any other feasible area.

$S_I \cap S_{II} = \emptyset$, there is no overlap in the feasible area.

3.3. Linear Programming with Interval Coefficient Solution

Algorithm 1General method to solve linear programming with interval coefficients: [9]
Let the following LPIC problem:

$$\text{Min } Z = \sum_{j=1}^{n} [\underline{c}_j, \overline{c}_j]\, x_j$$

subject to

$$\sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a}_{ij}]\, x_j \geq [\underline{b}_i, \overline{b}_i], \text{ for } i = 1, \dots, m$$

$x_j$is *sign-restricted*variable(that is$x_j \in W_I\ \forall j$)
Then the best optimum and the worst optimum as follows:

1. For The Best Optimum

$$\text{Min } \underline{\text{z}} = \sum_{j=1}^{n} c'_j x_j, \text{where} c'_j = \begin{cases} \underline{c}_j & , x_j \geq 0 \\ \overline{c}_j & , x_j \leq 0 \end{cases}$$

subject to

$$\sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \forall_i , \qquad \text{where} a'_{ij} = \begin{cases} \overline{a}_{ij} & , x_j \geq 0 \\ \underline{a}_{ij} & , x_j \leq 0 \end{cases} \qquad x_j \in W_I \forall_j .$$

2. For The Worst Optimum

$$\text{Min } \overline{\text{z}} = \sum_{j=1}^{n} c''_j x_j, \qquad \text{where} c''_j = \begin{cases} \overline{c}_j & , x_j \geq 0 \\ \underline{c}_j & , x_j \leq 0 \end{cases}$$

subject to

$$\sum_{j=1}^{n} a''_{ij} x_j \geq \overline{b}_i , \forall_i, \qquad \text{where} a''_{ij} = \begin{cases} \underline{a}_{ij} & , x_j \geq 0 \\ \overline{a}_{ij} & , x_j \leq 0 \end{cases} \qquad x_j \in W_I \forall_j .$$

$W_I$is variable set that is associated with interval coefficient.
If constraints have boundaries($\leq$)then that constraints multiple with (-1) to get ($\geq$).

Linear Progamming in Algorithm 1 have three possible results, that is: (i) optimum finite bounded point; (ii) unboundedness; or (iii) infeasibility, consequently LPIC have some possibilities as follows: [5]

a. If the best optimum is infeasible solution, then all LPIC are infeasible.
b. If the worst optimum unbounded solution, then all LPIC are unbounded
c. If best optimum solution feasible with value$\underline{z}$and worst optimum infeasible, then LPIC optimum has range in$\underline{z}$and infeasibility.
d. If the worst optimum solution is feasible with value$\overline{z}$and best optimum unbounded, then LPIC optimum has range between$-\infty$ and $\overline{z}$.

3.4. Numerical Examples

The following is an example of minimization which shows the case where the best optimum and the worst optimum is feasible.

Solve LPIC minimization model with inequality constraints as follows:

Min $Z = [1,3]x_1 + [2,4]x_2$

Subject to:

$C_1: [2,3]x_1 + [4,6]x_2 \geq [6,9]$, $C_2: x_1 + [2,4]x_2 \geq 5$,
$C_3: -x_1 + x_2 \geq [-2,-1]$, $C_4: [3,5]x_1 + x_2 \geq [6,7]$
$C_5: x_2 \leq 4$, $x_1, x_2 \geq 0$

Solution: Using Algorithm2.1, will get:

a. Best Optimum Solution:

Min $\underline{z} = x_1 + 2x_2$

Subject to:

$C_{1a}: 3x_1 + 6x_2 \geq 6$, $C_{2a}: x_1 + 4x_2 \geq 5$, $C_{3a}: -x_1 + x_2 \geq -2$
$C_{4a}: 5x_1 + x_2 \geq 6$, $C_5: x_2 \leq 4$, $x_1, x_2 \geq 0$

Solution for the Linear Programming model,then: $x_1 = 1$, $x_2 = 1$ dan $\underline{z} = 3$.

b. Worst Optimum Solution

Min $\overline{z} = 3x_1 + 4x_2$

Subject to:

$C_{1b}: 2x_1 + 4x_2 \geq 9$, $C_{2b}: x_1 + 2x_2 \geq 5$, $C_{3b}: -x_1 + x_2 \geq -1$
$C_{4b}: 3x_1 + x_2 \geq 7$, $C_5: x_2 \leq 4$, $x_1, x_2 \geq 0$

Solution for the linear programming model, then: $x_1 = 1{,}8$, $x_2 = 1{,}6$ dan $\overline{z} = 11{,}8$.

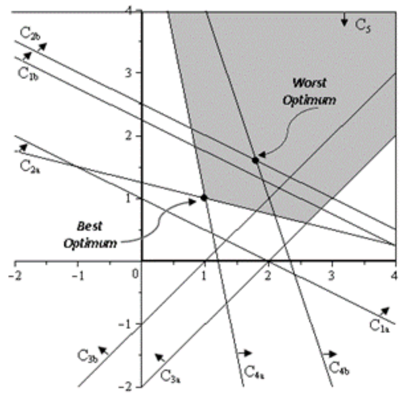

Figure 1. LPIC Feasible Area

Therefore, by using the best and worst optimum solution obtained before, then gained the upper bound and lower bound for the optimum value of the main problems. In this case the optimum value will take place between 3 and 11,8; that is $Z = [\,3, 11.8\,]$

The best and the worst optimum solution are illustrated in Figure 1.

So the optimum value will range between 3 and 11.8.

Various extreme versions of interval relationships can be obtained by fixing the interval coefficients at different combinations of their upper and lower limits. If any of the interval coefficients is fixed at an intermediate value, then an intermediate version of the relationship is obtained. The methods developed later in this research make special use of specific extreme versions of the constraints and objective.

Depending on the specific values chosen for the interval coefficients in the constraints, there is an infinite number of different possible feasible regions. In fact, some choices of coefficient values may render the Linear Programming infeasible or unbounded. Furthermore, if the objective function includes interval coefficients, there is also an infinite choice of objective functions. The LPIC goal of finding the best and worst optimum values are directly affected by which specific values are chosen for the interval coefficients, since these in turn determine the feasible regions and objective directions for specific versions of the model [1].

4. Conclusion

In the linear programming problem, sometimes coefficients in the model cannot be determined precisely so that is usually made in the form of estimation. One method to resolve this issue is to use the interval approach, where the indeterminate coefficients transformed into the interval. This linear programming form is called Linear Programming with Interval Coefficient (LPIC). Interval coefficient indicates expansion the tolerance interval (or area) where the constant parameters acceptable and meets the LPIC models. Completion of linear programming with interval coefficients with inequality constraints can be resolved with the following general steps:

Suppose given problem LPIC:

$$\text{Min } Z = \sum_{j=1}^{n} [\, \underline{c}_j, \overline{c}_j] \, x_j$$

subject to

$$Z = \sum_{j=1}^{n} [\, \underline{a}_{ij}, \overline{a}_{ij}] \, x_j \geq [\, \underline{b}_i, \overline{b}_i], \text{for} i = 1, \ldots, m$$

$x_j$is sign-restricted variable(that is$x_j \in W_I \; \forall j$)

Then best optimum and worst optimum as follows:

1. For The Best Optimum

$$\text{Min } \underline{\text{z}} = \sum_{j=1}^{n} c'_j x_j, \text{where} c'_j = \begin{cases} \underline{c}_j & , x_j \geq 0 \\ \overline{c}_j & , x_j \leq 0 \end{cases}$$

subject to

$$\sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \forall_i , \qquad \text{where} a'_{ij} = \begin{cases} \overline{a}_{ij} & , x_j \geq 0 \\ \underline{a}_{ij} & , x_j \leq 0 \end{cases} \qquad x_j \in W_I \forall_j .$$

2. For The Worst Optimum

$$\text{Min } \overline{\text{z}} = \sum_{j=1}^{n} c''_j x_j, \qquad \text{where} c''_j = \begin{cases} \overline{c}_j & , x_j \geq 0 \\ \underline{c}_j & , x_j \leq 0 \end{cases}$$

Subject to

$$\sum_{j=1}^{n} a''_{ij} x_j \geq \overline{b}_i , \forall_i \qquad \text{where} a''_{ij} = \begin{cases} \underline{a}_{ij} & , x_j \geq 0 \\ \overline{a}_{ij} & , x_j \leq 0 \end{cases} \qquad x_j \in W_I \forall_j .$$

By using best optimum and worst optimum solution then obtained upper bound and lower bound for LPIC optimum value. If constraints have boundaries (≤) then the constraints multiple by (-1) to get (≥).

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IOP Conf. Series: Materials Science and Engineering **288** (2017) 012061 doi:10.1088/1757-899X/288/1/012061

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