The Sociomathematical Norms in Linear Algebra Lecture

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Abstract: Social interaction between students through discussions in solving linear algebra problems is needed. It is because the linear algebra problem requires various strategies and is open-ended. This is called the sociomatematic norm. This study aims to analyze sociomatematic norms in Linear Algebra lectures in terms of two aspects namely sociomatematic norms associated with solving linear algebra problem and sociomatematic norms associated with participation in joint activities to solve linear algebra problem. The method used is descriptive-quantitative method. The subject of the research is the students of Mathematics Education UIN Sunan Gunung Djati Bandung, semester 3 academic year 2016/2017, as many as 120 students. The result of the research concludes that: sociomatic norms related to problem solving of good category Linear Algebra (70,52%), this is seen from problem solving strategy used by students in solving open problems; sociomatematic norms associated with participation in joint activities to solve the problem of linear Algebra category enough and well, all indicators including sociomatematic norms are implemented.

1 INTRODUCTION

Understanding Linear Algebra requires a number of thinking skills such as the ability to communicate mathematically and problem solving. Communication is the most important part in learning mathematics. (NCTM, 2000) emphasizes the importance of communication skills in mathematics and mathematics education. This must be supported through a planned and constructed classroom atmosphere so that students have the opportunity to interact and collaborate with each other (Tatsis and Koleza, 2008). In addition, the provision of a number of challenging issues to be solved through open-ended strategies will support the interaction between students (NCTM, 2000).

According to (Tatsis and Koleza, 2008) the interaction is contained in the teaching of mathematics more on aspects of analysis, so that many researchers who study issues related to learning mathematics. Each class as a group that interacts with certain interactions and behavior patterns will affect the quality of student learning (Zembat, I. O. and Yasa, 2015). In the classroom, norms are regular patterns of behavior that affect the nature of learning that occurs in them (Zoest, Stockero and Taylor, 2011).

The formation of social norms in the mathematics class aims to enable students to understand their role in the discussion. Students are not only required to speak out in response to math problems but are also required to analyze, criticize, and make solutions together, especially in terms of their mathematical reasoning (Roy, Tobias and Dixon, 2014). These activities are known as sociomatematic norms. Sociomatematics according to (Wedege, 2003) is a relationship between individuals, mathematics, and society in which there is activity numeracy, analysis and etnomatematics. While sociomatic norms are rules that apply to an interaction between students in solving problems, argue math and negotiate in understanding the concept of mathematics.

Sociomatematic norms evolved in the process of interaction and mutual participation during the course of mathematics learning. This is closely related to the negotiation and collective agreement on the application of relevant, appropriate, or different problem-solving procedures in problem solving and communicating the idea of settlement and way of thinking (Yacke, E and Cobb, 1996; Kang and Kim, 2016; Lopez, LM and Allal, 2007). This study aims to analyze the sociomatematic norms in the Linear Algebra lecture from two aspects, namely

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sociomatematic norms related to solving linear algebra problem and sociomatematic norms related to participation in joint activity to solve linear algebra problem.

2 METHODOLOGY

The subjects in this study are students of semester 3 academic year 2016/2017 who took courses Linear Algebra about 120 people, mathematics education program, UIN Sunan Gunung Djati Bandung, Indonesia. Linear Algebraic topics studied in this research are Linear Independence and Basis. The topic has a very wide scope and requires analysis and construction of a strong mathematical understanding.

2.1 Classroom Organization

Learning strategies are implemented through small group discussions followed by class discussions to solve linear algebra problems. Previously the lecturer introduced the rules to be used. Lecturers are actively involved during small group activities, including involvement in encouraging collaborative dialogue and dialogue and discussion of problem-solving efforts (Zembat, I. O. and Yasa, 2015). This research will analyze in depth how problem solving process of Linear Independence, Basis and student activity in group discussion.

Students are encouraged to solve problems through arguments and mathematical reasoning to obtain solutions with various strategies through the problem-solving process recommended by (Polya, 1973), namely: understanding the problem, devising a plan, carrying out the plan, and looking back. After students solve problems, the lecturers facilitate by setting the rules of reasoning verbalization and the mathematical strategies used to establish and maintain norms (Roy, Tobias and Dixon, 2014).

2.2 Research Instruments

The research instruments consist of observation sheets, student work results, homework and tests at the end of the lesson.

2.3 Data Collection Technique

The data collected includes observation and discussion activities in class. The linear algebra problem given is a challenging and open issue.

2.4 Data Analysis Technique

Linear Algebra problem-solving process based on Polya steps was analyzed quantitatively using percentage, while student discussion activity using sociomatematic norm indicator was analyzed descriptively.

3 RESULTS AND DISCUSSION

3.1 The Sociomathematical Norms Associated with Algebra Problem Solving

The following will describe the Linear Algebra problem-solving process through student work results. Examples of problems with the given Base topic:
The set of $S = \{v_1, v_2, v_3\}$ where vectors*:*
$v_1 = (2,1,-1)$, $v_2 = (-1,5,1)$ and $v_3 = (2,1,3)$. Investigate whether S is the basis for R^3?

Description of problem solving process done by students using problem-solving steps (Polya, 1973) as below:

3.1.1 Understanding the Problem

Results Analysis: Students can organize the structure of the problem they are facing through their writing about: a) what is known, b) what is asked, c) the problem condition to find what is being asked.
Achievement: 86.45%.

3.1.2 Devising a Plan

Analysis Result:

- Students can write down the definition of Basis, that is: *If V is any vector and S = {v1, v_2…., v_r} is a finite set of vectors in V, then S can be said to be a basis for V if: (i) S is linearly independent and (ii) S span V.* Achievement: 63,54%.
- Students can create mathematics model representative, that is:
  A vector equation that meets linear independence*, S= {v_1, v2, v3}* is linearly independent, then it must meet: $k_1v_1 + k_2 v_2 + k_3 v_3 = \mathbf{0}$ **and** requirement *S span* R^3 there is a consistent solution. Achievement: 50,78 %.

3.1.3 Carrying Out the Plan

Analysis Result:
Students solve problems with several strategies:

- Starting proof of S is linearly independent followed by $\boldsymbol{S}$ span V (1)
- Proof of S is linearly independent $\boldsymbol{S}$ span V done together through the coefficient matrix representation of the linear system form obtained from the linear independence and span description, as follows:

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{bmatrix} \quad (2)$$

Students using the strategy (1) take the following steps:

- Student prove *S is linearly independent and S span V* by row reduction by elementary row operation of a corresponding equation system. Achievement: 60.03%
- Students make conclusions based on the exploration of is linearly independent and S span V Achievement: 63.45%

Students using strategy (2) take steps:

- Students reorganize the structure of mathematical problems that compose, organize and develop it by looking for the determinant value of the coefficient matrix A.

Based on the results of calculations on the sociomatematic norms in Linear Algebra lectures related to linear Algebra problem solving are presented in the table as follows:

$$\det(A) = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{vmatrix} = 44$$

Achievement: 80,01%

- Students use the theorem to make conclusions based on the value of determinants that have been obtained. Achievement: 70,09%

3.1.4 Looking Back

Students perform the examinations that have been obtained. Achievement:

- Students using strategy (1) : 70,07%
- Students using strategy (2) : 70,01%

Based on the description of sociomatematic norms related to linear algebra problem solving are presented in Table 1.

Table 1: Achievement of linear algebra problem solving process.

Steps of Problem Solving (Polya, 1973)	Achievement (Percentage)	
	Strategy (1)	Strategy (2)
Understanding the problem	86,45%	86,45%
Devising a plan	57,16%	57,16%
Carrying out the plan	61,74%	75,05%
Looking back	70,07%	70,01%
Total	275,42 %	288,67 %
Average achievement of each strategy	68,86 %	72,17%
Average overall achievement	70,52 %	

The results of the analysis in Table 1 obtained the average of achievement of Linear Algebra problem-solving process using steps (Polya, 1973) of 70.52%. This is a good category.

3.2 The Sociomathematical Norms Associated with Participation in Joint Activities to Solve Linear Algebra Problems

Based on observations and interviews, the following describes the activities of one of the student discussions in small groups when solving the Linear Independence topic problem. Examples of problems as follows:

The set of vectors $S = \{v_1, v_2, v_3, v_4\}$, where $v_1 = (1,-1,1)$, $v_2 = (2, -2, 1)$, $v_3 = (3, -1, 1)$ and $v_4 = (2,1,-2)$. Investigate whether S is linearly independent!

Student A : Do we need to write down the definition completely ?
Student B : Yes, we need! so that we can easily make the path of completion

Both students start to write down the definition of Linear Independence, it is:
If $S = \{v_1, v_2, \ldots, v_r\}$ the set of vectors, then the vector equation: $k_1v_1+k_2v_2+ \ldots +k_rv_r=\boldsymbol{0}$ has only solution, namely $k1 = 0$, $k_2 = 0$, ..., $kr = 0$.

Based on that definition, the student starts exploring by first writing the vector equation: $k_1 v_1 + k_2 v_2 + \ldots + k_r v_r = \mathbf{0}$, then substituting the known vectors so that the system forms the following linear:

$$\begin{aligned} k_1 + 2\,k_2 + 3\,k_3 + 2\,k_4 &= 0 \\ -k_1 - 2\,k_2 - k_3 + k_4 &= 0 \\ k_1 + k_2 + k_3 - 2\,k_4 &= 0 \end{aligned} \quad (3)$$

Student A : Consider the form (3) do you have an opinion about the form?

Student B : Looking at the form (3) is somewhat different from what we have learned, here all the constants are 0, does the system have certain characteristics?

Sociosemantics Norm Aspect :
Students ask each other questions that emphasize the mathematical understanding

Lecturer activities:

- Providing assistance (scaffolding) is the provision of assistance in the early stages of guidance, encouragement, and describe the problem
- Providing direction to all groups to pay attention to form (3) and to recall material learned (Algebra Matrix) as a supporter of completion.
- Stimulating students' social interactions
- Directing the discussion on understanding the concept of Linear Algebra and its prerequisite concepts

Student A : I remember when learning Algebra Matrix, the form (3) is called the Homogeneous Linear Equation System but I forgot the practical way to solve the linear equations system of that form.

Student B : We have previously learned about the system of linear equations, what if we complete the form (3) with the Gauss-Jordan Elimination procedure.

Student A : OK! you work with Gauss-Jordan Elimination, I will study the shape characteristics (3)

Lecturer activities :

- Giving scaffolding by reminding material about form (3), whose solution is always *consistent (trivial and non trivial)*
- Recalls the theorem of the Homogeneous Linear Equation System.

Some students describe the theorem, namely: "The system of homogeneous linear equations with more unknown numbers (r) than the number of equations (n) then always has the number of solutions (non-trivial)"

Student B : I will finih it by using theorem

Sociomatematic Norm Aspects :
Students reach agreement using reasoning and mathematical proof to solve problems in different ways.

Student A : I have solved the system of linear equations (3) with the *Gauss-Jordan Elimination* obtained by the *non-trivial* settlement i.e. in addition to $k_1 = 0$, $k_2 = 0$, $k_3 = 0$ dan $k_4 = 0$ thus the conclusion *S is a linearly dependent*

Student B : By using the theorem it appears that the system of equation (3) has 4 unknown numbers ($r = 4$) and 3 equations ($n = 3$), since r is greater than n, then the form (3) has so many non- trivial) i.e. in addition to $k_1 = 0$, $k_2 = 0$, $k_3 = 0$ dan $k_4 = 0$thus the conclusion *S is a linearly dependent*

Lecturer activities :
Help students to draw on the conclusion

Sociomatematic Norm Aspects:

- Students explain the solutions they have using mathematical arguments
- Students compare their strategies to find mathematically important similarities and differences
- Students use mistakes as an opportunity to rethink the concepts of their mathematical ideas and test contradictions.

Based on the observation about sociomatematic norms related to participation in joint activity to solve linear algebra problem, the result obtained with fair and good category, it can be seen from student activity in small group discussion. Indicators of sociomatematic norms are always visible so as to assist the students in solving the given problems. Lecturer intervention appears through scaffolding. The summary is presented in Table 2.

Tabel 2: Sociothematic Norm related to students' participation in sharing activity.

Sociosemantic Norm Indicator	Category
1) Students ask each other questions that emphasize reasoning and mathematical understanding	Good
2) Students explain the solutions they have using mathematical arguments	Enough
3) Students reach agreement using reasoning and mathematical proof	Good
4) Students compare their strategies to find mathematically important similarities and differences	Good
5) Students use mistakes as an opportunity to rethink the concepts of their mathematical ideas and test contradictions.	Enough

Based on table 2 it can be concluded that the sociomatematic norms associated with participation in joint activities to solve the problem of linear Algebra category is fair and good.

3.3 Discussion

The results showed that the sociomatematic norms associated with solving linear algebra problems were good with 70.52% achievement. This can be seen from the completion steps done by students using Polya problem solving. Based on the analysis at the "understanding the problem" stage all students understand what is known, what is asked. Students can organize the problem structure.

At the "deceiving a plan" stage students apply some problem-solving strategies such as creating patterns, writing an equation, testing special cases or simpler cases of problems encountered to get a better picture of problem solving, and identifying parts of the whole.

At the "carrying out the plan" stage, students have been able to execute the strategy as planned in the previous stage, and at the "looking back" stage the student can check the results on the original problem. The sociomatematic norms associated with participation in joint activities to solve linear algebra problems resulted in quite good results. This is in line with the results of the research (Hurst et al., 2013) social interaction provides a means for students to view topics from different perspectives and improve their thinking, critical issues and problem-solving abilities. As according to (Tatsis, 2007) social norms and sociomatematic norms developed during interaction among students when working together to solve a mathematical problem and in presenting a mathematical solution result.

Open-ended problem solving supports the creation of sociomatematic norms. As an opinion (Capraro, M. M., Capraro, R. M and Cifarelli, 2007) that open-ended problem solving provides a free and supportive learning environment for students to develop and express their mathematical understanding.

4 CONCLUSIONS

Based on the results of the study and discussion it can be concluded that the sociomatematic norms associated with Algebra linear problem solving is in a good category (70.53%), it can be seen from the problem solving strategy used by students in solving the problems that are open-ended, students solve linear algebra problem with various strategies, including the use of definitions and theorems, so that students are not fixated on procedural settlement. Completion is done following the procedure (Polya, 1973); the sociomatematic norms associated with participation in joint activities to solve the Linear Algebra category problem are sufficient and good, all indicators including sociomatematic norms are implemented in all student discussion groups.

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