



Motorcycle's Trajectory Tracking Model Based on Polynomial Least-squares Approximation

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This paper provides a simple method for modeling the trajectory of motorcycle movements in mixed traffic condition. The trajectories model has been built using Polynomial Least-squares Approximation based on the coordinates that earned from the motorcycle maneuvers. In order to collect the data, this research uses video cameras fixed at elevated positions to record traffic flow. There are three steps of modeling motorcycle's trajectory: 1) data preprocessing to obtained pixel coordinate which represents motorcycle's trajectory, 2) coordinates transformation to convert the pixel coordinate to real-world coordinate, and 3) build a trajectory model using polynomial least-square approximation. This method was implemented for tracking the trajectory of motorcycle's maneuver. The maneuvers are when they try to avoid the collision with another motorcycle, car, and pedestrian. Based on experimental and analysis result for twenty samples of data, using a comparison of MSE for each degree of the polynomial, the trajectory model of motorcycle maneuver for avoiding an object in front of them can be described properly by 4th order polynomial equation. This result shows that trajectory model using polynomial least-squares approximation give an accurate outcome and also has the simpler computation. Furthermore, the conclusion of this study about trajectory tracking model for motorcycle may be instructive as a prior knowledge for building mixed traffic model based on their trajectory behaviors. This model also provides the information necessary for safe behavior decision making or motion planning.

Keywords: Polynomial Least-squares, Modeling, Vehicle's Trajectory, Movement's Model

1. INTRODUCTION

Today, many models for describing vehicle movement have been developed. This model is very important to build a traffic model that can describe the phenomenon of the real traffic. Over more than half a century, traffic flow theorists have been pursuing two goals: 1) simple

and efficient models to abstract vehicular traffic flow and 2) a unified framework in which existing traffic flow models fit and relate to each other¹.

Continuing this effort, there are many kinds of the literature of traffic flow theories and models have been developed, but researchers generally agree that modeling has not yet reached a satisfying level. Studies on traffic flow become important since many reasons behind that, such as: 1) it is necessary to develop traffic model which

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can describe the real phenomena; 2) traffic model can support for developing intelligent transportation system (ITS) which related with safety driving system; and 3) traffic model support for future issue about intelligent car².

To make a traffic model closed to realistic, we should have much information about the behavior of vehicles. In mixed traffic condition, different vehicles will have different behavior. There are many ways and methods to obtain information about the behavior of a vehicle, such as using traffic simulator, doing a deep interview with the driver and spreading the questionnaire to the user. Besides that, the trajectory tracking could be one of the methods to get some information about vehicle's behavior. Vehicle's trajectory can provide much information about the movement of vehicles that can be used to analyze driving behavior and traffic safety³.

The issue of modeling vehicle trajectory has been widely addressed in the recent years. In mixed traffic environments, occlusions and false detections are common and although there have been substantial advances in the last year, tracking is still a challenging task. So, the motion trajectory learning has still become an important research topic, a few attempts have been made to study this issue.

Tracking is often divided into two steps: detection which is finding the objects of interest on every frame, and data association which is matching the detections to form a complete trajectory in time⁴. In 2010, Rucco et al.⁵ try to explore the trajectory of the vehicle by using novel nonlinear optimal control techniques. In this research, Rucco et al. computed aggressive trajectories of the car vehicle and studied the vehicle's behavior depending on its parameters. The result has shown that there are many of the interesting dynamics effects of a real car. In order to reduce the computational complexity of inference process, Gindele et al.⁶ using probabilistic methods built a framework for estimating driver behaviors and vehicle trajectories in traffic environments. The evaluation of this framework has been tested for highway traffic scenarios. The result has shown a good performance although there is still much noise in the testing process. Lately, some researchers have been developed the vehicle trajectory models using image processing methods⁷, such as Gaussian Mixture Model⁸ and Super Pixel Tracking Methods⁹.

In this study, we proposed methods for modeling the tracking trajectory based on polynomial least-square approximation. Based on some literature, least square methods is quick, stable and insensitive to small errors in the data^{10,11}, which very useful to represent the trajectory model of vehicle in mathematical equations. Besides that, most of the previous studies on vehicle trajectories focused only on common traffic which is a homogenous condition. So, the main objective of this study is to build a motorcycle's trajectory model using a polynomial approximation in mixed traffic condition particularly.

The paper is organized as follows. In Section 2, we briefly review about polynomial least-square approximation to find the coefficient of a polynomial. Modeling the trajectory of motorcycle's movement using

polynomial approximation is presented in Section 3. The experimental result and analysis of applying the model to track the trajectory of a motorcycle in urban traffic condition are shown in Section 4. Finally, in Section 5 we conclude this paper by noting that the polynomial approximation could be implemented to tracking the motorcycle's movement trajectory for certain maneuver.

2. POLYNOMIAL LEAST-SQUARES APPROXIMATION

Mathematical modeling is one of the powerful methods for describing the phenomena in the real world. Polynomial data fitting can be solved by least-squares methods. The least-square method is a mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offset of the points from the curve. Furthermore, least-squares fitting methods is a fundamental problem in many aspects, such as pattern recognition, computer vision, graphics and medical imaging analysis. In this section, the principle and basic notation of polynomial equation using least square method will be explained.

Let $[x_k, y_k] \forall k \in N$ be dispersion point in R^2 . A general regression polynomial is given by:

$$\begin{aligned} P_n(x) &= c_1 + c_2x + c_3x^2 + \dots + c_{n+1}x^n \\ P_n(x) &= \sum_{k=0}^n c_{k+1}x^k \end{aligned} \quad (1)$$

Where $f_1 = 1, f_2 = x, f_3 = x^2$, etc. According to the Least-square principle, the coefficient c_k can be determined by:

$$[c_k] = \left[\sum_{k=1}^n f_i(x_k) \cdot f_j(x_k) \right]_{n \times n}^{-1} \cdot \left[\sum_{k=1}^n f_j(x_k) \cdot y_k \right]_{n \times 1} \quad (2)$$

After we found the coefficient c_k , the polynomial equation can be obtained, and the mean square error (MSE) can be calculated.

3. TRACKING MOTORCYCLE'S TRAJECTORY-BASED ON POLYNOMIAL APPROXIMATION

In order to build a mixed traffic in the urban condition which is very close to the real phenomena, we should describe and represent details behavior of each driver on the microscopic level. The motion of the driver can be seen from their trajectory. This trajectory correlated with the interaction made between the vehicles. Furthermore, this trajectory also could be a prior knowledge to build a behavior model for mixed traffic particularly.

Data for microscopic traffic behavior studies are very crucial since it is used for the basic kinematic parameters of vehicles (speed, direction, acceleration and deceleration) and also for analyze the multi-vehicular interactions (following distance and speed difference)¹³. The vehicle's trajectory could be obtained manually from video sequences. This method is naturalistic observation which ensures that the normal behavior can be observed

and the data collected are not affected by the presence of researchers.

Based on the video footage, we can choose what maneuver we will be investigated. Firstly, the video footage of the traffic stream has to be digitalized and be converted into an AVI (Audio Video Interleave) file. By using software, we can see the images frame by frame according to the given video frame processing interval to tracking the motorcycle's trajectories manually. The screen photo coordinates are marked using manual clicking by the mouse which then converted to the real-world coordinates and after that the model of motorcycle's trajectories can be obtained using a polynomial approximation.

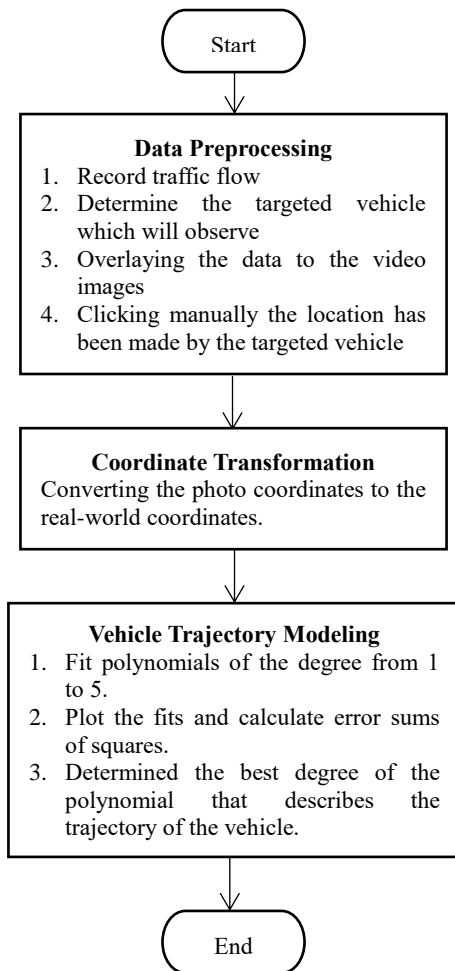


Fig. 1. Trajectory modeling sequence

Based on Figure 1 there are three main processes of modeling motorcycle's trajectory: data preprocessing to obtained pixel coordinate which represents motorcycle's trajectory, coordinates transformation to convert the pixel coordinate to real-world coordinate, and the last step is to build a trajectory model using the polynomial least-square approximation.

Data preprocessing is the first step to finding pixel coordinate which represents the motion of targeted vehicle we will observe. In this research, we built a simple program to process the video footage in order to get the coordinates point of vehicle's motion. There are three steps to obtain the coordinates photo of motorcycle's

motion as we seen in a screenshot in Figure 2. The first step is to determine the targeted motorcycle to see their trajectory. Then, to get the trajectory of targeted vehicle, we do the overlay data to the video images. The last step is clicking manually the location has been made by the targeted vehicle on the monitor by using a mouse. Then, the program will show us the path of pixel coordinates of the vehicle, so that the pixel coordinates can be obtained. The flow of data preprocessing can be seen more clearly in Figure 2 as follows.



Fig. 2. The flow of data preprocessing, (a) Targeted vehicle; (b) Maneuver's trajectory; (c) Photo coordinate in a pixel.

Coordinates transformation is useful to convert photo coordinate into real-world coordinate. We need real-world coordinate which describes as if that camera could display the data coordinate from a top-view angle. Hence, the trajectory movements of targeted vehicle can then be examined without the effects of perspective. We can use mathematical approach for mapping the coordinates between the real-world and the video image, by using matrix transformation¹⁵.

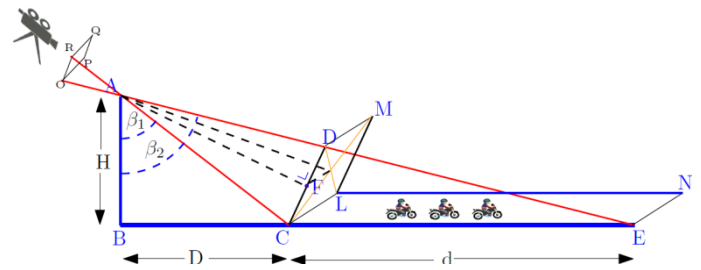


Fig. 3. Geometry illustration of camera view

Based on Figure 3, a video camera was located at fixed elevated position to record the traffic flow on the road $CENL$, with CE is the length of the road (meters) and CL is the width of the roads (meters). Side $CLDM$ is a virtual side which is perpendicular to the camera, and $OPQR$ is the picture image that caught in camera (pixels). So, the coordinate transformation from pixel to real-world coordinate is done by two processes of transformation. Firstly, transform pixel coordinate (x, y) in $OPQR$ side to perpendicular side $CLDM$ which represent actual perpendicular coordinate (x', y') using matrix transformation M_1 . Secondly, transform the actual perpendicular coordinates in $CLDM$ (x', y') to real-world coordinate (x'', y'') in $CENL$ side using matrix transformation M_2 . Finally, we can find general matrix transformation for converting photo coordinates to real-world coordinate by multiplying matrices M_1 with M_2 .

Using mathematical approach [15], the general matrix transformation M which convert pixel coordinate to real

world coordinates was presented in Equation 4 and Equation 5.

$$\begin{bmatrix} x_{real} \\ y_{real} \end{bmatrix} = M \begin{bmatrix} x_{photo} \\ y_{photo} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} x_{real} \\ y_{real} \end{bmatrix} = M_1 M_2 \begin{bmatrix} x_{photo} \\ y_{photo} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} x_{real} \\ y_{real} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & 0 \\ 0 & k_1 \end{bmatrix} \begin{bmatrix} s/b & 0 \\ 0 & r/a \end{bmatrix} \begin{bmatrix} x_{photo} \\ y_{photo} \end{bmatrix}, y' < \frac{r}{2} \\ \begin{bmatrix} 1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} s/b & 0 \\ 0 & r/a \end{bmatrix} \begin{bmatrix} x_{photo} \\ y_{photo} \end{bmatrix}, y' \geq \frac{r}{2} \end{cases}$$

where,

$$k_1 = \frac{\sin \left[\tan^{-1} \frac{AF}{FH} \right]}{\sin \left[\tan^{-1} \frac{AB}{BC} + \tan^{-1} \frac{AF}{CF} - \tan^{-1} \frac{AF}{FH} \right]}$$

$$k_2 = \frac{\sin \left[\tan^{-1} \frac{AB}{BC} + \tan^{-1} \frac{AF}{CF} \right]}{\sin \left[\tan^{-1} \frac{AB}{BC} + \tan^{-1} \frac{AF}{CF} - \tan^{-1} \frac{AF}{FJ} - 180 \right]}$$

$$r = 2\sqrt{H^2 + D^2} \sin \left[\frac{\tan^{-1} \frac{D+d}{H} - \tan^{-1} \frac{D}{H}}{2} \right]$$

s : the width of the roads (meters)

a, b : pixels camera

(x_{real}, y_{real}) is the real-world coordinate data and (x_{photo}, y_{photo}) is the photo coordinate data.

Modeling the vehicle's trajectory is the last step to obtain the trajectory model for motorcycle's maneuver in mixed urban traffic. In order to obtain the optimal curve model which proper with the actual trajectory, we used polynomial approximation and computed by least-squares methods. The motivation of using least-square methods is it can give the simplest computation and accurate outcome to track vehicle's trajectory^{12,14}.

This study will focus on modeling the maneuver of motorcycle's trajectory that tries to avoid collision with another vehicle (car, motorcycle, and pedestrian). We perform the polynomial fitting for degree one to five based on the real-world coordinate that obtained from the previous step. After that, we compare the polynomial of 1st to 5th order based on curve plotting and the error sums of square of each order to find the optimum degree which can properly describe the actual trajectory.

4. EXPERIMENTAL AND ANALYSIS RESULT

This section will give the experimental and analysis result to verify the effectiveness of our methods to track the trajectory of the motorcycle in mixed traffic condition. Our approach will be implemented to track motorcycle's maneuver. There are three maneuvers will be modeling in this section: 1) motorcycle's maneuver when they try to avoid pedestrian; 2) motorcycle's maneuver when they try to avoid car; 3) motorcycle's maneuver when they try to avoid another motorcycle. The data was collected from three different locations which recorded in peak hours. In order to model the motorcycle's trajectory, we use twenty

samples data for each case of maneuver. The twenty data samples here is divided into two characteristics, the first one is the sample that showed maneuver the vehicle to avoid an object without being influenced by surrounding vehicle (traffic conditions are not so crowded), and ten sample data remainder is maneuvering the vehicle to avoid an object on crowd traffic conditions.



Fig. 4. Sample data of motorcycle's maneuver avoiding object: (a) avoid pedestrian; (b) avoid car; (c) avoid motorcycle.

In this research, we use polynomial least-squares approximation for modeling the trajectory of maneuver based on the coordinate we have earned before. We use ten coordinates that describe the trajectory of motorcycle's maneuver, fit and plot it into a polynomial of 1st to 5th order. The example of plotting coordinates of motorcycle maneuver can be seen in Figure 5.

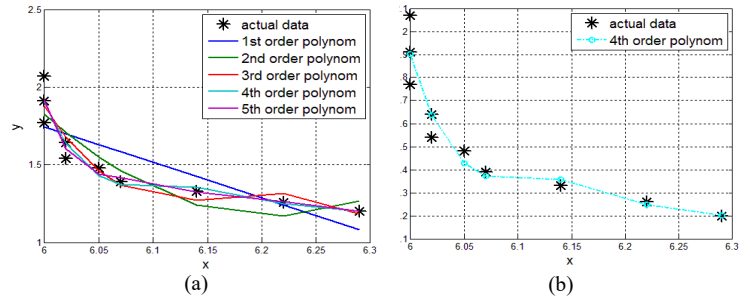


Fig. 5. Plotting coordinates: (a) Comparison of 1st to 5th order polynomial trajectory; (b) 4th order polynomial trajectory

TABLE I. Mean Square Error of Polynomial Trajectory

Maneuver	Degree of Polynomial				
	1 th	2 nd	3 rd	4 th	5 th
Avoid car	0.4736	0.2758	0.2169	0.1867	0.1867
Avoid car*	1.9768	0.6520	0.5748	0.4911	0.2993
Avoid motorcycle	1.3431	0.0762	0.0388	0.0241	0.0241
Avoid motorcycle*	0.4320	0.1680	0.0973	0.0781	0.0781
Avoid pedestrian	0.3708	0.1783	0.1495	0.1484	0.1484
Avoid pedestrian*	0.4429	0.2823	0.2351	0.2124	0.2124

*in crowd condition

Based on Figure 5 and Table 1, the 4th order of polynomial gives the best and optimum curve fitting for modeling the trajectory of the motorcycle. The MSE shows the improvement in conjunction with increasing polynomial degree and becomes optimum in 4th order polynomial. We can see there are no much differences MSE between the first ten data sample with the last one. The results show that trajectory model using Polynomial Least-squares Approximation can give the accurate outcome. This information could be important for some

research as a prior knowledge of analysis about the behavior of the vehicle and modeling the traffic, especially in urban traffic condition.

4. CONCLUSIONS

In this paper, a simple trajectory model approach based on Polynomial Least-squares Approximation was introduced, and the approach was applied based on motorcycle maneuver track which extracted from the video footage. This study only performs the modeling of motorcycle maneuvers when they try to avoid collision with a car, pedestrian, and another motorcycle.

Based on experimental and analysis result for twenty samples of data, using a comparison of MSE for each degree of the polynomial, the trajectory model of motorcycle maneuver for avoiding an object in front of them can be described properly by 4th order polynomial equation. This result shows that trajectory model using polynomial least-squares approximation give an accurate outcome and also has the simpler computation.

Furthermore, the conclusion about trajectory tracking model for a motorcycle is a preliminary study which useful as a prior knowledge for building mixed traffic model based on their behaviors. And also, this model provides the information necessary for safe behavior decision making or motion planning in the traffic system.

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REFERENCES

- [1] J. Wang, L. Zhang, D. Zhang, and K. Li. An adaptive longitudinal driving assistance system based on driver characteristic, *IEEE Transactions on Intelligent Transportation System*, 14(1)(2013) 1-12.
- [2] R. Mardiyati, N. Ismail, and A. Faruqi. Review of a microscopic model for traffic flow, *ARNP Journal of Engineering and Applied Sciences*, 9(10)(2014) 1794-1800.
- [3] G. Lu, B. Cheng, Y. Wang, and Q. Lin. A car-following model based on quantified homeostatic risk perception, *Mathematical Problems in Engineering*, (2013).
- [4] L. Leal-Taixe and B. Rosenhahn. Modeling, simulation and visual analysis of crowds: A Multidisciplinary Perspective, ser. *The International Series in Video Computing*, Eds. Springer Science & Business Media, (2013).
- [5] A. Rucco, G. Notarstefano, and J. Hauser. Dynamics exploration of a single-track rigid car model with load transfer, in *49th IEEE Conference on Decision and Control (CDC)*, (2010) 4934-4939.
- [6] T. Gindele, S. Brechtel, and R. Dillmann. A probabilistic model for estimating driver behaviors and vehicle trajectories in traffic environments, in *Intelligent Transportation System (ITSC)*, 2010, *13th International IEEE Conference on*, (2010) 1625-1631.
- [7] I. Pathan and C. Chauhan. A survey on moving object detection and tracking methods, *(IJCSIT) International Journal of Computer Science and Information Technologies*, 6(6)(2015).
- [8] Z. Chen and T. Ellis. Automatic lane detection from vehicle motion trajectories, in *Advanced Video and Signal Based Surveillance (AVSS)*, 2013 *10th IEEE International Conference*, (2013) 466-471.
- [9] D. T. Lin and C. H. Hsu. Crossroad traffic surveillance using superpixel tracking and vehicle trajectory analysis, in *Pattern Recognition (ICPR)*, 2014 *22nd International Conference on*, Aug 2014, pp. 2251-2256.
- [10] Q. Li and J. G. Griffiths. Least squares ellipsoid specific fitting, in *Geometric Modeling and Processing, Proceedings*, (2004) 335-340.
- [11] M. Goldhammer, S. Khler, K. Doll, and B. Sick. Camera based pedestrian path prediction by means of polynomial least-squares approximation and multilayer perceptron neural networks, in *SAI Intelligent Systems Conference (IntellSys)*, 2015, Nov 2015, pp. 390-399.
- [12] X. Ying, L. Yang, J. Kong, Y. Hou, S. Guan, and H. Zha. Direct least square fitting of ellipsoids, in *Pattern Recognition (ICPR)*, 2012 *21st International Conference on*, (2012) 3228-3231.
- [13] J. W. P. Tzu-Chang Lee and M. Bell. *Trajectory extractor user manual version 1.0*, (2008).
- [14] M. Pilu, A.W. Fitzgibbon, and R. B. Fisher. Ellipse-specific direct least-square fitting, in *Image Processing, 1996, Proceedings, International Conference on*, 3(1996) 599-602.
- [15] R. Mardiyati, I. Maryono, E. Mulyana, and K. Usman. Matrix transformation from pixel coordinate to real-world coordinate for vehicle trajectory using mathematical approach, *International Journal on Advanced Science, Engineering and Information Technology*, (7)(2017).