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A novel double-convection chaotic attractor, its adaptive control and circuit simulation

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Abstract. A 3-D novel double-convection chaotic system with three nonlinearities is proposed in this research work. The dynamical properties of the new chaotic system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, stability analysis of equilibria, etc. Adaptive control and synchronization of the new chaotic system with unknown parameters are achieved via nonlinear controllers and the results are established using Lyapunov stability theory. Furthermore, an electronic circuit realization of the new 3-D novel chaotic system is presented in detail. Finally, the circuit experimental results of the 3-D novel chaotic attractor show agreement with the numerical simulations.

1. Introduction

A chaotic system is commonly defined as a nonlinear dynamical system that is highly sensitive to even small perturbations in its initial conditions [1-4]. In other words, a chaotic system is a nonlinear dynamical system with at least one positive Lyapunov exponent. In the last two decades, many new chaotic systems have been discovered such as Tigan system [5], Li system [6], Jafari system [7], Sundarapandian systems [8-9], Pehlivan system [10], Vaidyanathan systems [11-14], Molaie systems [15], Tacha system [16], Sampath system [17], Volos system [18], Wang system [19], Pham systems [20-22], etc.

Chaos theory has several applications in science and engineering such as oscillators [23-25], chemical reactors [26-28], Tokamak systems [29], voice encryption [30], population biology [31-32], robotics [33-34], neural networks [35-36], memristors [37-38], secure communication system [39-41] etc.

The problem of chaos control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [1-4]. The problem of synchronization of two chaotic systems deals with synchronizing identical state trajectories of a pair of chaotic systems called master and slave systems asymptotically [1-4]. Some popular methods for chaos control and synchronization of chaotic systems can be listed as an active control, adaptive control, backstepping control, fuzzy control, sliding mode control etc., which are outlined in [1-4].

Rucklidge chaotic system is a popular model in mechanics for nonlinear double convection [42]. When the convection takes place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modeled by Rucklidge's 3-D system of differential equations, which produces chaotic solutions.

In this work, we modify the dynamics of Rucklidge system [42] and derive a new double-convection chaotic system with an absolute nonlinearity and two quadratic nonlinearities. The phase portraits of the new chaotic system are displayed in Section 2, and the dynamical properties of the new chaotic system such as dissipativity, equilibria analysis, Lyapunov exponents, Kaplan-Yorke dimension, etc. are analyzed in Section 3. Lyapunov exponents of the new chaotic system are obtained as $L_1 = 0.4684$, $L_2 = 0$ and $L_3 = -3.6684$. The Kaplan-Yorke dimension of the new system is found as $D_{KY} = 2.1277$.

Adaptive control and synchronization of the new chaotic system with unknown system parameters are discussed in Sections 4 and 5, respectively. The main adaptive control results derived in this work are established using Lyapunov stability theory [43]. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 6. The circuit experimental results of the new chaotic attractor show agreement with the numerical simulations. Section 7 contains the conclusions.

2. A new nonlinear double-convection chaotic system

In fluid mechanics modeling, cases of two-dimensional convection in a horizontal layer of Boussinesq fluid with lateral constraints were considered by Rucklidge [42]. When the convection takes place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modeled by a new third-order set of ordinary differential equations, which produces chaotic solutions.

The Rucklidge chaotic system is described by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2 x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants. In [42], it was established that the system (1) is chaotic for $a = 2.2$ and $b = 6.7$.

In this work, we modify the dynamics of Rucklidge chaotic system (1) and obtain a new dynamics for nonlinear double convection as

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - |x_2| - x_2 x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{2}$$

where x_1, x_2, x_3 are state variables and a, b are positive constants.

In this paper, we show that the system (2) is *chaotic* for the parameter values

$$a = 2.2, \quad b = 18 \tag{3}$$

For numerical simulations, we take the initial values of the system (2) as

$$x_1(0) = 0.2, \quad x_2(0) = 0.2, \quad x_3(0) = 0.2 \tag{4}$$

Figure 1 shows the phase portraits strange attractor of the new double-convection chaotic system (2) for the parameter values (3) and initial conditions (4). Figure 1 (a) shows the 3-D phase portrait of the nonlinear double-convection chaotic system (2). Figures 1 (b)-(c) show the projections of the strange attractor of the nonlinear double-convection chaotic system (2) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively.
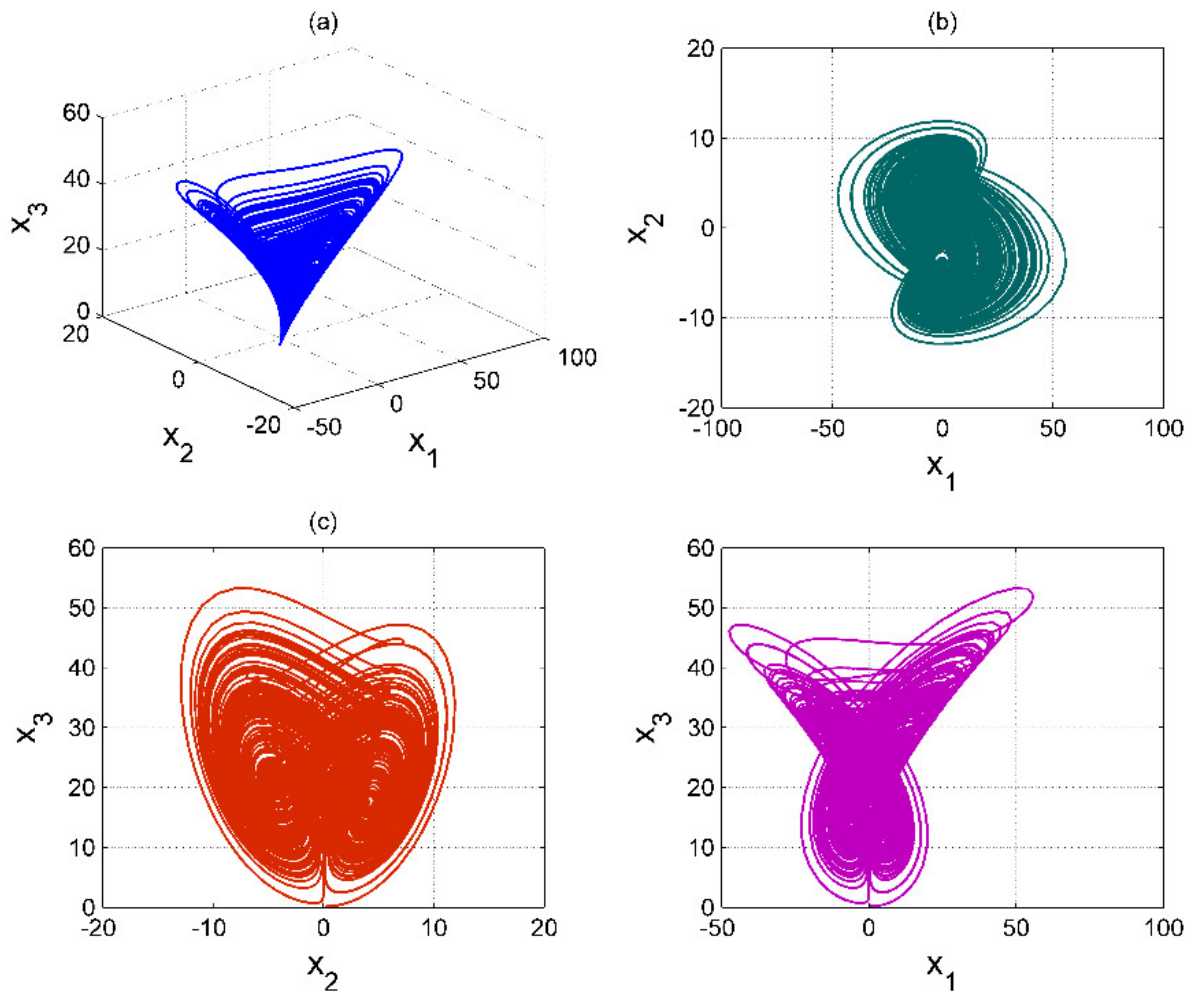

Figure 1. Phase portraits of the new chaotic system (2) for $a = 2.2$, $b = 18$

3. Dynamical properties of the new chaotic system
In this section, we take the parameter values as in the chaotic case, *i.e.* $a = 2.2$ and $b = 18$.

3.1. Dissipativity
If V denotes any volume along the flow of the new chaotic system (2), then

$$\nabla \cdot V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -a - 1 = -\mu < 0 \tag{5}$$

where $\mu = a + 1 > 0$.

This shows that $\dot{V} = -\mu V$. Integrating, we get

$$V(t) = \exp(-\mu t) V(0) \tag{6}$$

Thus, the new chaotic system (2) is dissipative. Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (2) settles into a strange attractor of the system.

3.2. Equilibrium points

The equilibrium points of the system (2) are obtained by solving the system of equations

$$-ax_1 + bx_2 - x_2 x_3 = 0, \quad x_1 = 0, \quad -x_3 + x_2^2 = 0 \tag{7}$$

We take the parameter values as in the chaotic case (3), *i.e.* $a = 2.2$ and $b = 18$.

A simple calculation shows that the new chaotic system (2) has three equilibrium points given by

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 0 \\ 4.1231 \\ 17 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0 \\ -4.3589 \\ 19 \end{bmatrix} \tag{8}$$

To check the stability of the equilibrium points, we calculate the Jacobian of the system (2) as

$$J(x) = \begin{bmatrix} -a & b - \mathrm{sign}(x_2) - x_3 & -x_2 \\ 1 & 0 & 0 \\ 0 & 2x_2 & -1 \end{bmatrix} \tag{9}$$

Let $J_0 = J(E_0)$, $J_1 = J(E_1)$ and $J_2 = J(E_2)$. We find that J_0 has the eigenvalues

$$\lambda_1 = -1, \quad \lambda_2 = -5.4829, \quad \lambda_3 = 3.2829 \tag{10}$$

This shows that the equilibrium point E_0 is a saddle point. Hence, it is unstable.

Next, we find that J_1 has the eigenvalues

$$\lambda_1 = -4.4335, \quad \lambda_{2,3} = 0.6168 \pm 2.6997i \tag{11}$$

This shows that the equilibrium point E_1 is a saddle-focus. Hence, it is unstable.

We also find that J_2 has the eigenvalues

$$\lambda_1 = -4.5512, \quad \lambda_{2,3} = 0.6756 \pm 2.8095i \tag{12}$$

This shows that the equilibrium point E_2 is a saddle-focus. Hence, it is unstable.

3.3. Lyapunov exponents and Kaplan-Yorke dimension

The parameters of the new system (2) are taken as $a = 2.2$ and $b = 18$. The initial state of the system (2) is taken as $x_i(0) = 0.2$ for $i = 1, 2, 3$.

The Lyapunov exponents of the system (2) are calculated using Wolf's algorithm [44] (see Figure 2) as

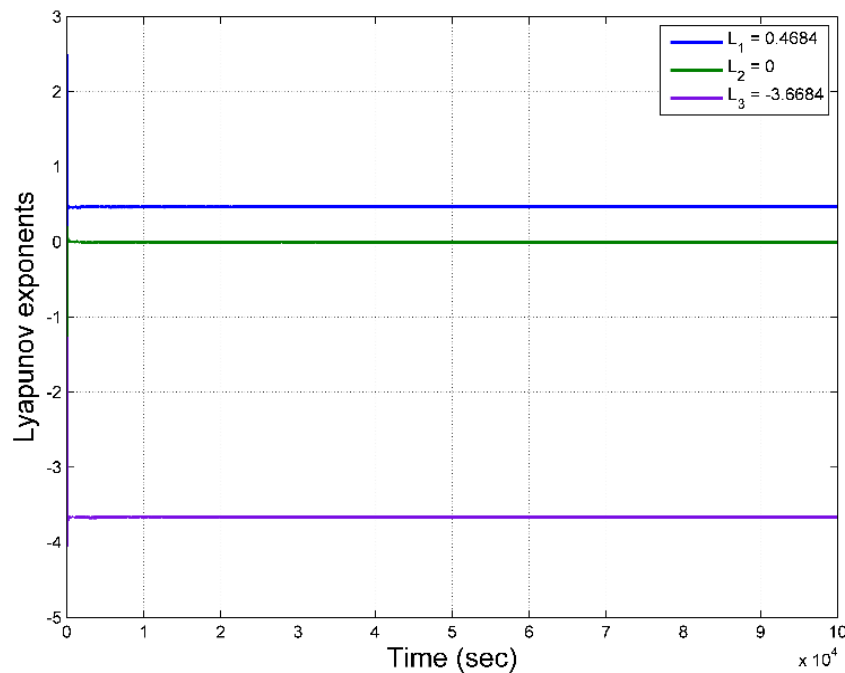$$L_1 = 0.4684, \quad L_2 = 0, \quad L_3 = -3.6684. \tag{13}$$

Figure 2. Lyapunov exponents of the new chaotic system (2)

The Kaplan-Yorke dimension of the new chaotic system (2) is calculated as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1277 \tag{14}$$

3.4. Symmetry and invariance

The new chaotic system (2) is invariant under the coordinate transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{15}$$

This shows that the chaotic system (2) has a rotation symmetry about the x_3-axis. Hence, every non-trivial trajectory of the system (2) must have a twin trajectory.

Also, the x_3-axis is invariant under the flow of the new chaotic system (2). This invariant flow is characterized by the 1-D dynamics $\dot{x}_3 = -x_3$ which is exponentially stable.

4. Adaptive control of the new chaotic system

In this section, we consider the controlled new chaotic system given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - |x_2| - x_2x_3 + u_1 \\ \dot{x}_2 = x_1 + u_2 \\ \dot{x}_3 = -x_3 + x_2^2 + u_3 \end{cases} \tag{16}$$

where x_1, x_2, x_3 are the states and a, b are unknown system parameters.

We consider the adaptive controller defined by

$$\begin{cases} u_1 = \hat{a}(t)x_1 - \hat{b}(t)x_2 + |x_2| + x_2x_3 - k_1x_1 \\ u_2 = -x_1 - k_2x_2 \\ u_3 = x_3 - x_2^2 - k_3x_3 \end{cases} \tag{17}$$

where $\hat{a}(t), \hat{b}(t)$ are estimates of a, b respectively and k_1, k_2, k_3 are positive constants.

Substituting (17) into (16), we obtain the closed-loop system

$$\begin{cases} \dot{x}_1 = -[a - \hat{a}(t)]x_1 + [b - \hat{b}(t)]x_2 - k_1 x_1 \\ \dot{x}_2 = -k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{18}$$

We define parameter estimation errors as follows:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{19}$$

Using (19), the closed loop system (18) reduces to

$$\begin{cases} \dot{x}_1 = -e_a x_1 + e_b x_2 - k_1 x_1 \\ \dot{x}_2 = -k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{20}$$

Differentiating (19) with respect to t, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{21}$$

Next, we consider the Lyapunov function defined by

$$V\left(x_1, x_2, x_3, e_a, e_b\right) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2\right) \tag{22}$$

which is positive definite on R^5.

Differentiating V along the trajectories of (20) and (21), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left(-x_1^2 - \dot{\hat{a}}\right) + e_b\left(x_1 x_2 - \dot{\hat{b}}\right) \tag{23}$$

In view of (23), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}} = -x_1^2 \\ \dot{\hat{b}} = x_1 x_2 \end{cases} \tag{24}$$

Next, we prove the main theorem of this section.

Theorem 1. The new chaotic system (16) with unknown parameters is globally and asymptotically stabilized by the adaptive control law (17) and the parameter update law (24), where k_1, k_2, k_3 are positive constants.

Proof. The Lyapunov function V defined by (22) is quadratic and positive definite on R^5.

By substituting the parameter update law (24) into (23), we obtain the time-derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{25}$$

which is negative semi-definite on R^5.

Thus, by Barbalat's lemma [43], it follows that the closed-loop system (20) is globally asymptotically stable for all initial conditions $x(0) \in R^3$.

Hence, we conclude that the new chaotic system (16) with unknown parameters is globally and asymptotically stabilized by the adaptive control law (17) and the parameter update law (24), where k_1, k_2, k_3 are positive constants. This completes the proof.   ∎

For numerical simulations, we take the gain constants as $k_i = 10$ for $i = 1, 2, 3$.

We take the parameter values as in the chaotic case (3), i.e. $a = 2.2$ and $b = 18$.

We take the initial conditions of the states of the new chaotic system (16) as $x_1(0) = 18.3$, $x_2(0) = 11.7$ and $x_3(0) = 16.4$.

We take the initial conditions of the parameter estimates as $\hat{a}(0) = 7.5$ and $\hat{b}(0) = 6.8$.

Figure 3 shows the time-history of the states of the new chaotic system (16) after the implementation of the adaptive control law (17) and the parameter update law (24).
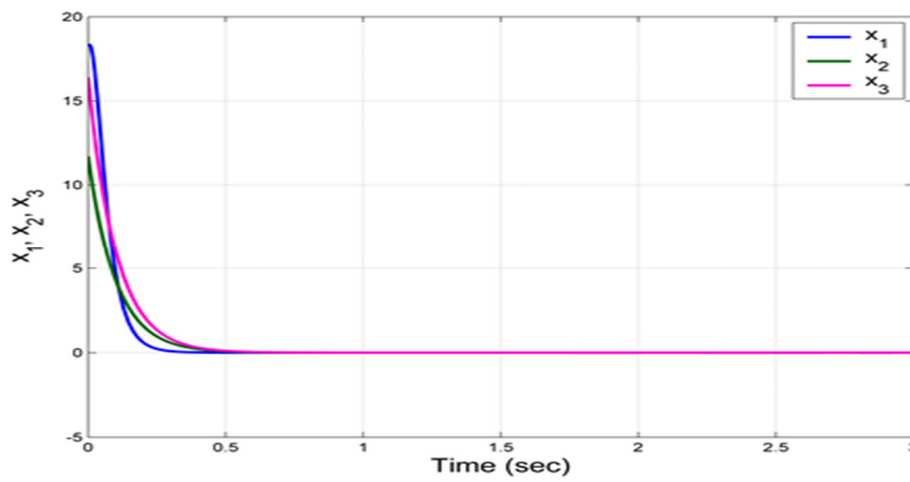

Figure 3. Time-history of the controlled state trajectories of the new chaotic system

5. Adaptive synchronization of the new chaotic system

In this section, we use the adaptive control to synchronize a pair of identical new chaotic systems with unknown state parameters.

As the master system, we consider the new chaotic system given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - |x_2| - x_2 x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{26}$$

where x_1, x_2, x_3 are the states and a, b are unknown parameters.

As the slave system, we consider the new chaotic system given by

$$\begin{cases} \dot{y}_1 = -ay_1 + by_2 - |y_2| - y_2 y_3 + u_1 \\ \dot{y}_2 = y_1 + u_2 \\ \dot{y}_3 = -y_3 + y_2^2 + u_3 \end{cases} \tag{27}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be designed.

The synchronization error between the systems (26) and (27) is defined as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{28}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = -ae_1 + be_2 - |\,y_2\,| + |\,x_2\,| - y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 = e_1 + u_2 \\ \dot{e}_3 = -e_3 + y_2^2 - x_2^2 + u_3 \end{cases} \tag{29}$$

We consider the adaptive control defined by

$$\begin{cases} u_1 = \hat{a}(t)e_1 - \hat{b}(t)e_2 + |\,y_2\,| - |\,x_2\,| + y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 = -e_1 - k_2 e_2 \\ u_3 = e_3 - y_2^2 + x_2^2 - k_3 e_3 \end{cases} \tag{30}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (30) into (29), we obtain the closed-loop system

$$\begin{cases} \dot{e}_1 = -[a - \hat{a}(t)]e_1 + [b - \hat{b}(t)]e_2 - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{31}$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{32}$$

Using (32), we can simplify (31) as

$$\begin{cases} \dot{e}_1 = -e_a e_1 + e_b e_2 - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{33}$$

Differentiating (32) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{34}$$

Next, we consider the Lyapunov function defined by

$$V\left(e_1, e_2, e_3, e_a, e_b\right) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \tag{35}$$

which is positive definite on R^5.

Differentiating V along the trajectories of (33) and (34), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left(-e_1^2 - \dot{\hat{a}}\right) + e_b\left(e_1 e_2 - \dot{\hat{b}}\right) \tag{36}$$

In view of (36), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}} = -e_1^2 \\ \dot{\hat{b}} = e_1 e_2 \end{cases} \tag{37}$$

Next, we prove the main theorem of this section.

Theorem 2. The new chaotic systems (26) and (27) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (30) and the parameter update law (37), where k_1, k_2, k_3 are positive constants.

Proof. The Lyapunov function V defined by (35) is quadratic and positive definite on R^5.

By substituting the parameter update law (37) into (36), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{38}$$

which is negative semi-definite on R^5.

Thus, by Barbalat's lemma [43], it follows that the closed-loop system (33) is globally asymptotically stable for all initial conditions $e(0) \in R^3$.

Hence, we conclude that the new chaotic systems (26) and (27) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (30) and the parameter update law (37), where k_1, k_2, k_3 are positive constants. This completes the proof. ∎

For numerical simulations, we take the gain constants as $k_i = 10$ for $i = 1, 2, 3$.

We take the parameter values as in the chaotic case (3), i.e. $a = 2.2$ and $b = 18$.

We take the initial conditions of the states of the master system (26) as $x_1(0) = 20.2$, $x_2(0) = 6.4$ and $x_3(0) = 15.1$.

We take the initial conditions of the states of the slave system (27) as $y_1(0) = 18.5$, $y_2(0) = 12.3$ and $y_3(0) = 10.7$.

We take the initial conditions of the parameter estimates as $\hat{a}(0) = 16.3$ and $\hat{b}(0) = 5.8$.

Figure 4 shows the synchronization of the states of the new chaotic systems (26) and (27).

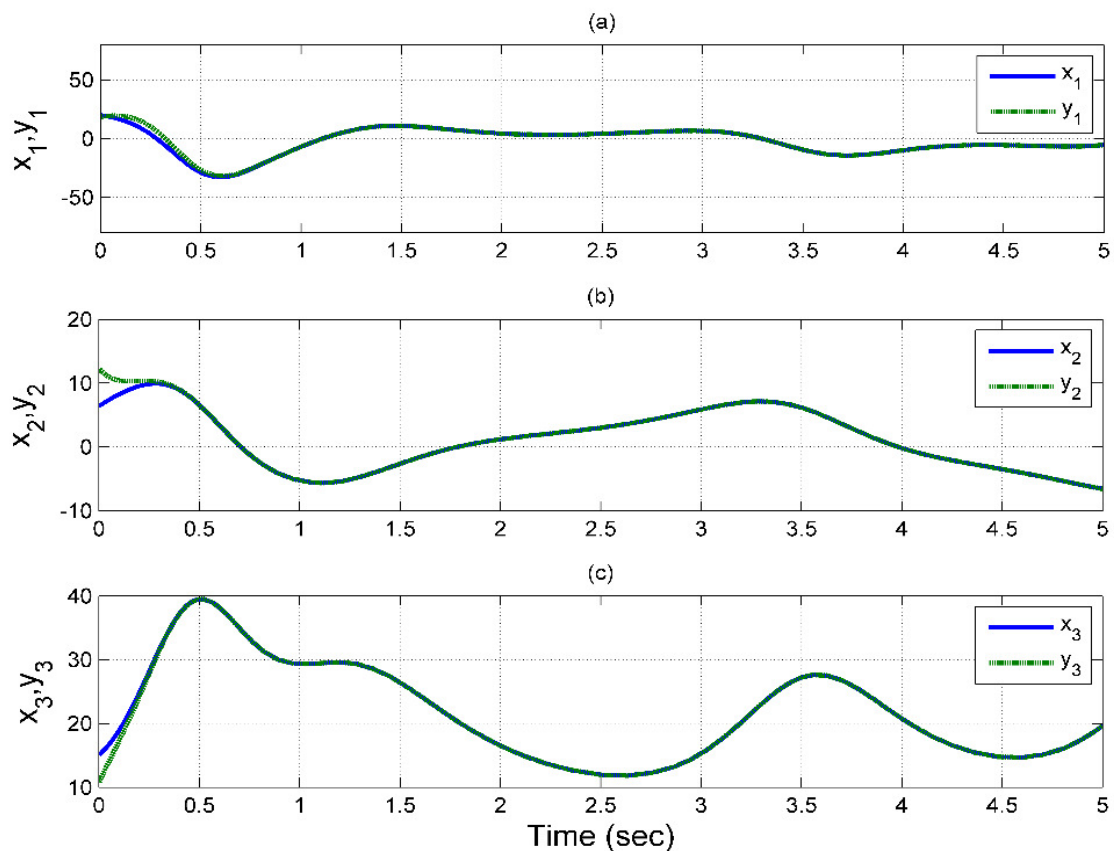Figure 5 shows the time-history of the synchronization errors e_1, e_2, e_3.

Figure 4. Complete synchronization of the new chaotic systems
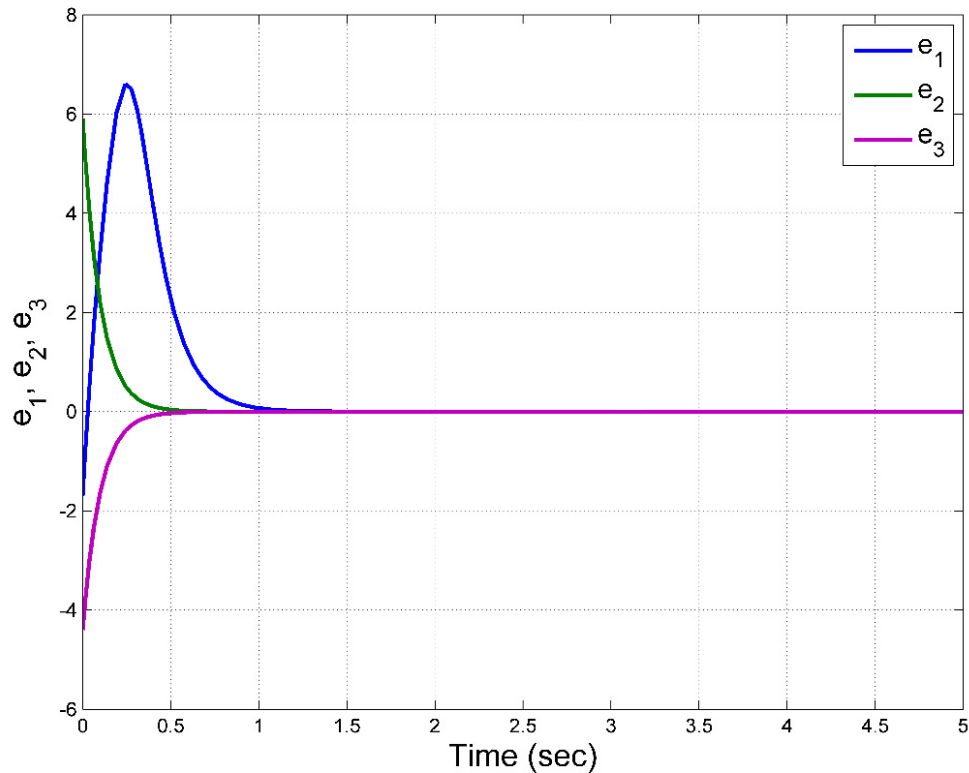
Figure 5. Time-history of the synchronization errors for the new chaotic systems

6. Circuit implementation of the new double-convection chaotic system

Electronic circuit provides an alternative approach to exploring new chaotic system (2). In this section, we design and build an electronic circuit of the system (2) as shown in Figure 6. In more details, there are three integrators (U1A, U3A, U5A), which are created by the operational amplifiers. The circuit consists of simple electronic elements, such as resistors, capacitors, operational amplifiers, analog devices AD633 multipliers and two diodes (1N4148), which provide the signal |X2|. By applying Kirchhoff's laws to the circuit in Figure 6, its circuital equations are derived in the following form:

$$
\begin{cases}
\dot{x}_1 = -\dfrac{1}{C_1 R_1} x_1 - \dfrac{1}{C_1 R_2} |x_2| + \dfrac{1}{C_1 R_3} x_2 - \dfrac{1}{10 C_1 R_4} x_2 x_3 \\[2mm]
\dot{x}_2 = \dfrac{1}{C_2 R_5} x_1 \\[2mm]
\dot{x}_3 = -\dfrac{1}{C_3 R_6} z + \dfrac{1}{10 C_3 R_7} x_2^{\,2}
\end{cases}
\tag{39}
$$

We choose the values of the circuital elements as

$$
\begin{cases}
R_1 = 4.54\,K\Omega,\ R_3 = 555\,\Omega,\ R_4 = R_7 = 1\,K\Omega \\[1mm]
R_2 = R_5 = R_6 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = 10\,K\Omega \\[1mm]
C_1 = C_2 = C_3 = 10\,nF
\end{cases}
\tag{40}
$$

The supplies of all active devices are ± 15 volt. The proposed circuit is implemented in the electronic simulation package MultiSIM. The obtained phase portraits are shown in Figure 1. There is a good agreement between these circuital results and the theoretical ones (see Figures. 7–9).

7. Conclusions

This work proposed a novel three-dimensional double-convection chaotic system with three nonlinearities. The dynamical properties of the new chaotic system were discussed in detail. The qualitative properties included phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, stability analysis of equilibria, etc. Adaptive control and synchronization of the new chaotic system with unknown parameters were achieved via nonlinear controllers and Lyapunov stability theory. Furthermore, an electronic circuit realization of the new 3-D novel chaotic system was proposed and the circuit experimental results of the 3-D novel chaotic attractor showed good agreement with the numerical simulations.
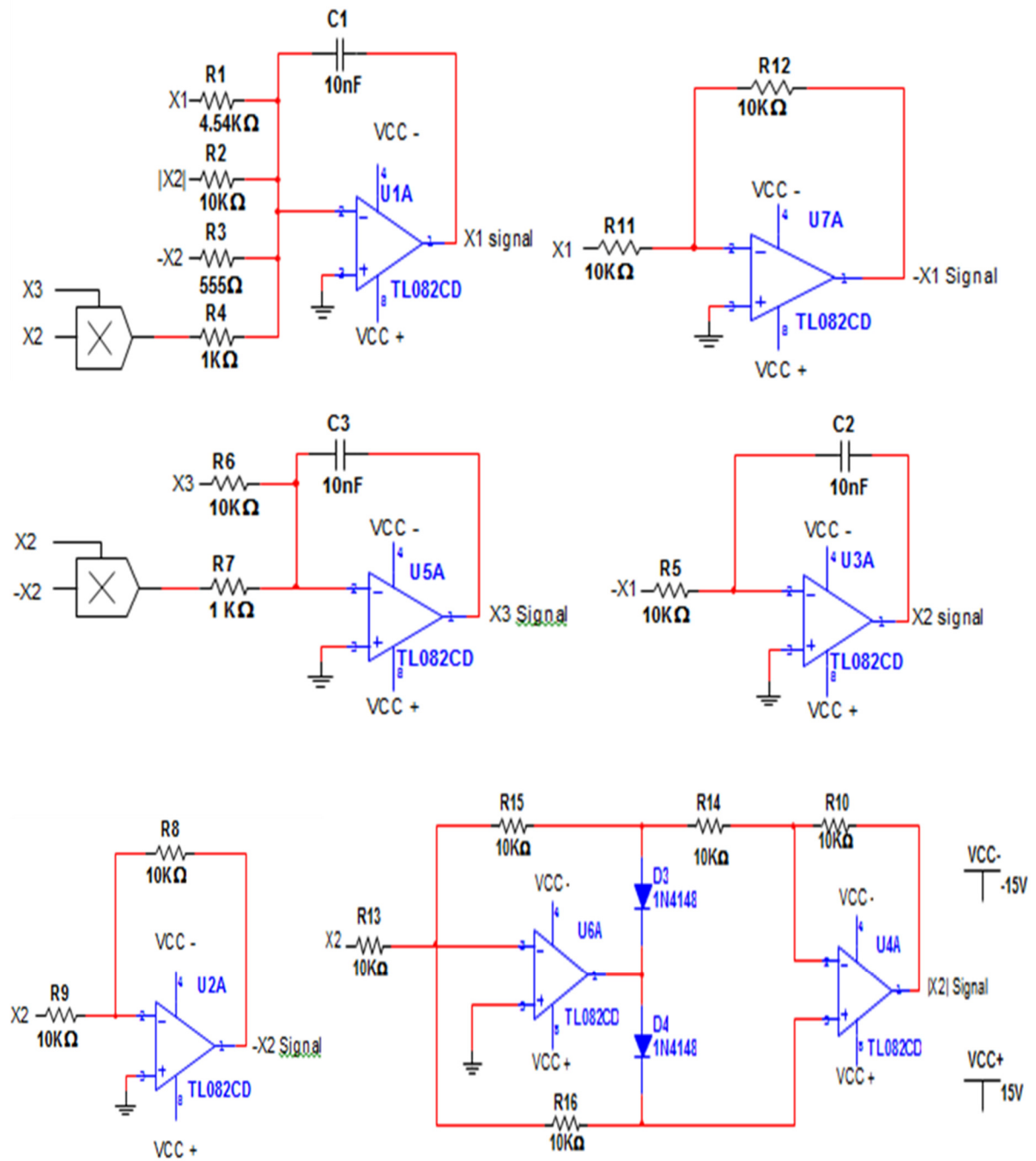
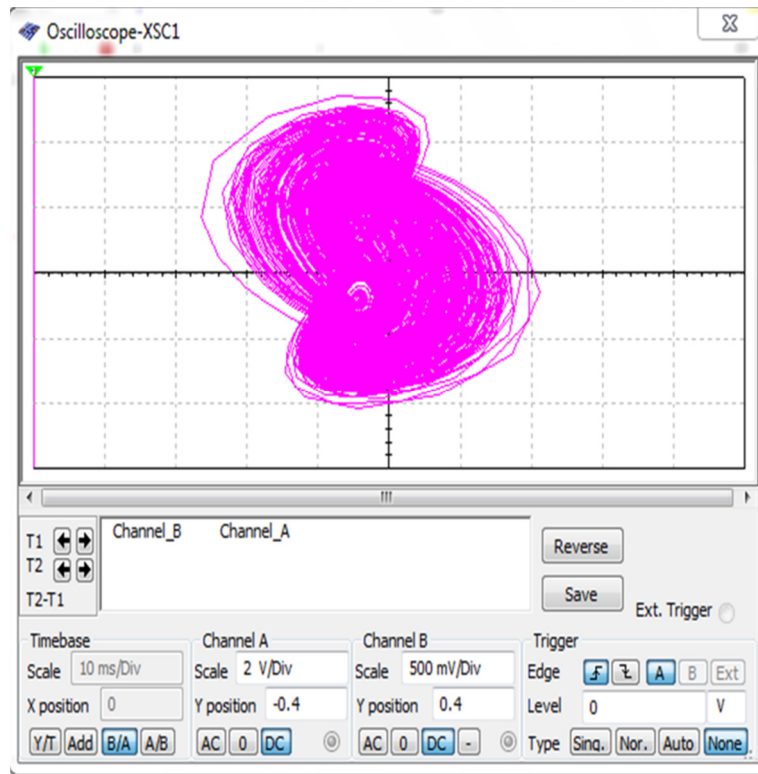**Figure 6** Schematic of the proposed new chaotic system by using MultiSIM

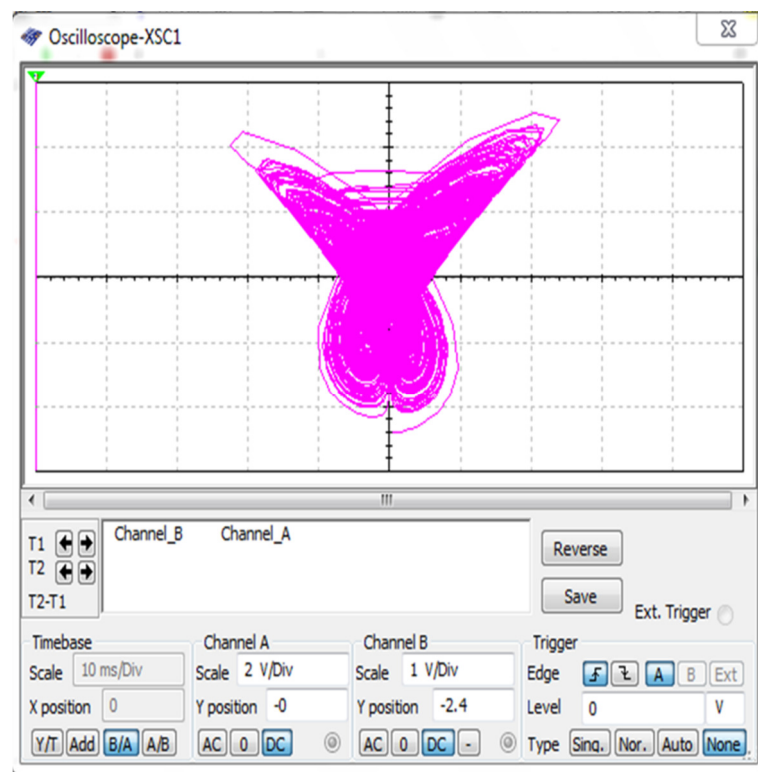**Figure 7** 2-D projection of the new chaotic system on the (x_1, x_2) plane

Figure 8 2-D projection of the new chaotic system on the (x_1, x_3) plane
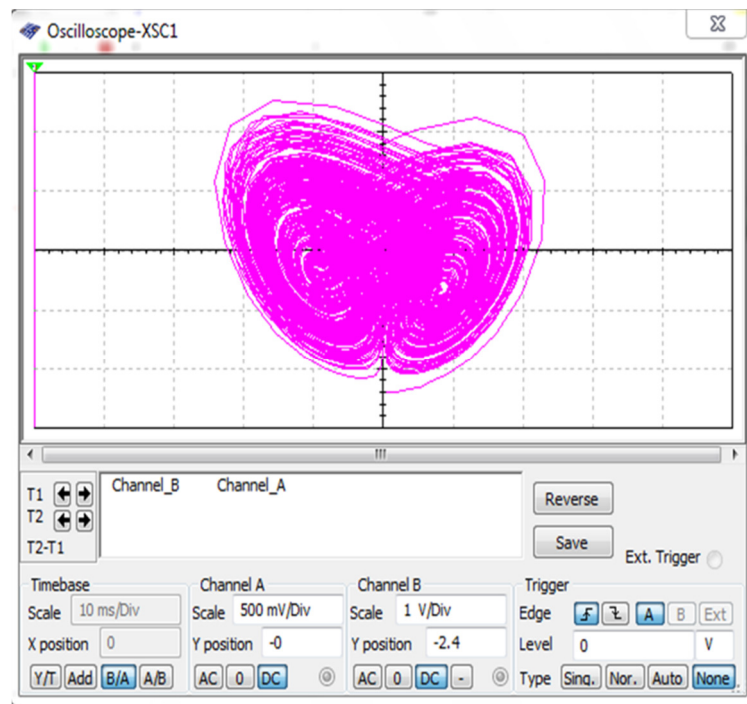
Figure 9 2-D projection of the new chaotic system on the (x_2, x_3) plane

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