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# A new chaotic attractor with two quadratic nonlinearities, its synchronization and circuit implementation

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**Abstract.** A 3-D new chaotic attractor with two quadratic nonlinearities is proposed in this paper. The dynamical properties of the new chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. We show that the new chaotic system has three unstable equilibrium points. The new chaotic attractor is dissipative in nature. As an engineering application, adaptive synchronization of identical new chaotic attractors is designed via nonlinear control and Lyapunov stability theory. Furthermore, an electronic circuit realization of the new chaotic attractor is presented in detail to confirm the feasibility of the theoretical chaotic attractor model.

## 1. Introduction

Chaos theory deals with nonlinear dynamical systems that are highly sensitive to initial conditions, which is also characterized by the existence of a positive Lyapunov chaos exponent [1-2]. In the last few decades, chaotic and hyperchaotic systems have attracted the interest of the engineering and science community because of their wide applications in many scientific and engineering fields [3-18].

Motivated by the research on chaotic systems with dissipative chaotic attractors with quadratic nonlinearities [19-24], we prove a new 3-D new chaotic system with two quadratic nonlinearities in this paper. The new chaotic system is dissipative and it has three unstable equilibrium points.

Section 2 describes the new chaotic system and details the properties such as Lyapunov exponents and Kaplan-Yorke dimension. Section 3 describes the adaptive synchronization of the new chaotic system with unknown parameters. Furthermore, an electronic circuit realization of the new chaotic



system is presented in detail in Section 4. The circuit experimental results of the new chaotic attractor show agreement with the numerical simulations. Section 5 contains the conclusions of this work.

## 2. A new chaotic system with two quadratic nonlinearities

In this paper, we announce a new 3-D chaotic system with two quadratic nonlinearities given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_3 \\ \dot{x}_2 = x_1 x_3 \\ \dot{x}_3 = 50 - b x_1^2 - x_3 \end{cases} \quad (1)$$

where  $x_1, x_2, x_3$  are state variables and  $a, b$  are positive constants.

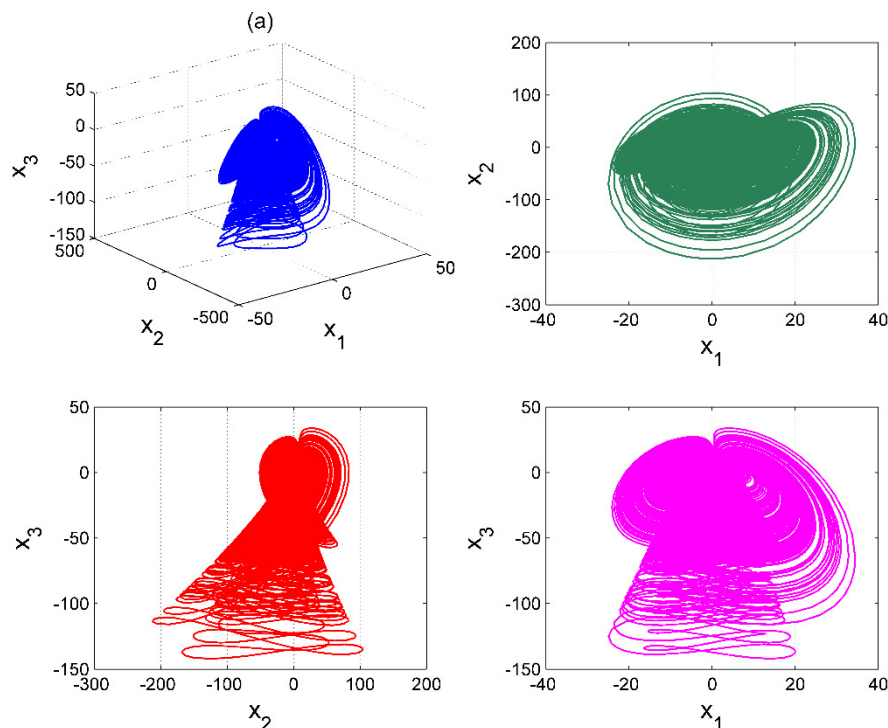
In this paper, we show that the system (1) is *chaotic* for the parameter values

$$a = 3, \quad b = 1 \quad (2)$$

For numerical simulations, we take the initial values of the system (1) as

$$x_1(0) = 0.2, \quad x_2(0) = 0.2, \quad x_3(0) = 0.2 \quad (3)$$

Figure 1 shows the phase portraits strange attractor of the new chaotic system (1) for the parameter values (2) and initial conditions (3). Figure 1 (a) shows the 3-D phase portrait of the new chaotic system (1). Figures 1 (b)-(c) show the projections of the new chaotic system (1) in  $(x_1, x_2)$ ,  $(x_2, x_3)$  and  $(x_1, x_3)$  coordinate planes, respectively.



**Figure 1.** Phase portraits of the new chaotic system (1) for  $a = 3, b = 1$

For the rest of this section, we take the parameter values as in the chaotic case (2). The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$a(x_2 - x_1) - x_3 = 0, \quad x_1 x_3 = 0, \quad 50 - b x_1^2 - x_3 = 0 \quad (4)$$

Solving the equations in (4) we obtain the equilibrium points of the system (1) as

$$E_1 = \begin{bmatrix} 0 \\ 50/3 \\ 50 \end{bmatrix}, \quad E_2 = \begin{bmatrix} \sqrt{50} \\ \sqrt{50} \\ 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} -\sqrt{50} \\ -\sqrt{50} \\ 0 \end{bmatrix} \tag{5}$$

It is easy to show that  $E_1$  is a saddle point, while  $E_2$  and  $E_3$  are saddle-focus points.

For the parameter values as in the chaotic case (2) and the initial state as in (3), the Lyapunov exponents of the new 3-D system (2) are determined using Wolf's algorithm as

$$L_1 = 1.0676, \quad L_2 = 0, \quad L_3 = -5.0676 \tag{6}$$

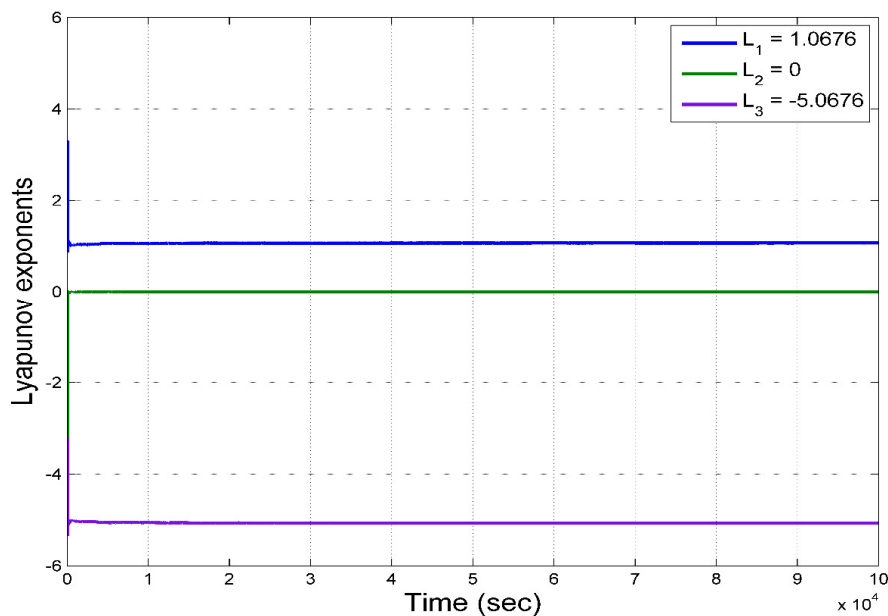
Since  $L_1 > 0$ , the new 3-D system (1) is chaotic. Thus, the system (1) exhibits a chaotic attractor. Also, we note that the sum of the Lyapunov exponents in (6) is negative. This shows that the new 3-D chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the new 3-D system (1) is determined as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2107, \tag{7}$$

which indicates the complexity of the new chaotic system (1).

Figure 2 shows the Lyapunov exponents of the new chaotic system (1) with a strange attractor.



**Figure 2.** Lyapunov exponents of the new chaotic system (1) for  $a = 3, b = 1$

### 3. Adaptive synchronization of the new chaotic systems

In this section, we devise adaptive controller so as to synchronize the respective states of identical new chaotic systems with unknown parameters considered as *master* and *slave* systems respectively. As the master system, we consider the new chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_3 \\ \dot{x}_2 = x_1 x_3 \\ \dot{x}_3 = 50 - bx_1^2 - x_3 \end{cases} \tag{8}$$

where  $x_1, x_2, x_3$  are the states and  $a, b$  are unknown parameters.

As the slave system, we consider the new chaotic system given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) - y_3 + u_1 \\ \dot{y}_2 = y_1 y_3 + u_2 \\ \dot{y}_3 = 50 - b y_1^2 - y_3 + u_3 \end{cases} \quad (9)$$

where  $y_1, y_2, y_3$  are the states and  $u_1, u_2, u_3$  are adaptive controls to be designed.

The synchronization error between the systems (8) and (9) is defined as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \quad (10)$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) - e_3 + u_1 \\ \dot{e}_2 = y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 = -b(y_1^2 - x_1^2) - e_3 + u_3 \end{cases} \quad (11)$$

We consider the adaptive control defined by

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) + e_3 - k_1 e_1 \\ u_2 = -y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3 = \hat{b}(t)(y_1^2 - x_1^2) + e_3 - k_3 e_3 \end{cases} \quad (12)$$

where  $k_1, k_2, k_3$  are positive gain constants.

Substituting (12) into (11), we obtain the closed-loop system

$$\begin{cases} \dot{e}_1 = [a - \hat{a}(t)](e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -[b - \hat{b}(t)](y_1^2 - x_1^2) - k_3 e_3 \end{cases} \quad (13)$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \quad (14)$$

Using (14), we can simplify (13) as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -e_b(y_1^2 - x_1^2) - k_3 e_3 \end{cases} \quad (15)$$

Differentiating (14) with respect to  $t$ , we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \quad (16)$$

Next, we consider the Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2) \quad (17)$$

which is positive definite on  $\mathbf{R}^5$ .

Differentiating  $V$  along the trajectories of (15) and (16), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a [e_1(e_2 - e_1) - \hat{a}] + e_b [-e_3(y_1^2 - x_1^2) - \hat{b}] \quad (18)$$

In view of (18), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}} = e_1(e_2 - e_1) \\ \dot{\hat{b}} = -e_3(y_1^2 - x_1^2) \end{cases} \quad (19)$$

Next, we prove the main theorem of this section.

**Theorem 2.** The new chaotic systems (8) and (9) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (12) and the parameter update law (19), where  $k_1, k_2, k_3$  are positive constants.

*Proof.* The Lyapunov function  $V$  defined by (17) is quadratic and positive definite on  $\mathbf{R}^5$ .

By substituting the parameter update law (19) into (18), we obtain the time-derivative of  $V$  as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \quad (20)$$

which is negative semi-definite on  $\mathbf{R}^5$ .

Thus, by Barbalat's lemma [25], it follows that the closed-loop system (15) is globally asymptotically stable for all initial conditions  $e(0) \in \mathbf{R}^3$ .

Hence, we conclude that the new chaotic systems (8) and (9) with unknown parameters are globally and asymptotically stabilized by the adaptive control law (12) and the parameter update law (19), where  $k_1, k_2, k_3$  are positive constants.

This completes the proof. ■

For numerical simulations, we take the gain constants as

$$k_1 = 10, \quad k_2 = 10, \quad k_3 = 10 \quad (21)$$

We take the parameter values as in the chaotic case (2), i.e.

$$a = 3, \quad b = 1 \quad (22)$$

We take the initial conditions of the states of the master system (8) as

$$x_1(0) = 5.2, \quad x_2(0) = 8.9, \quad x_3(0) = 3.1 \quad (23)$$

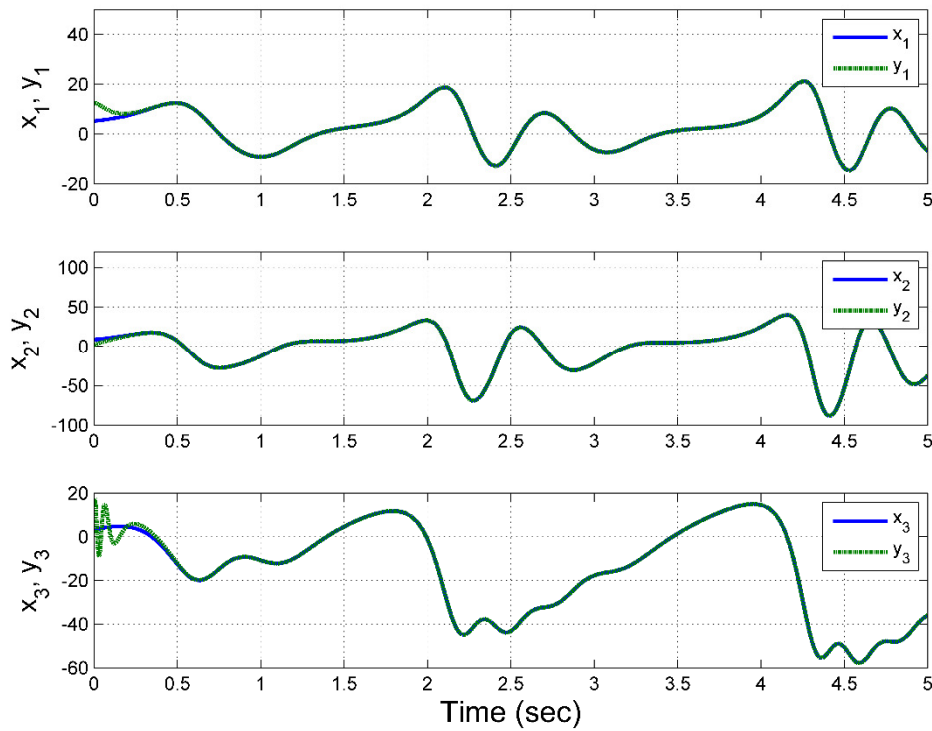
We take the initial conditions of the states of the slave system (9) as

$$y_1(0) = 12.5, \quad y_2(0) = 2.1, \quad y_3(0) = 14.7 \quad (24)$$

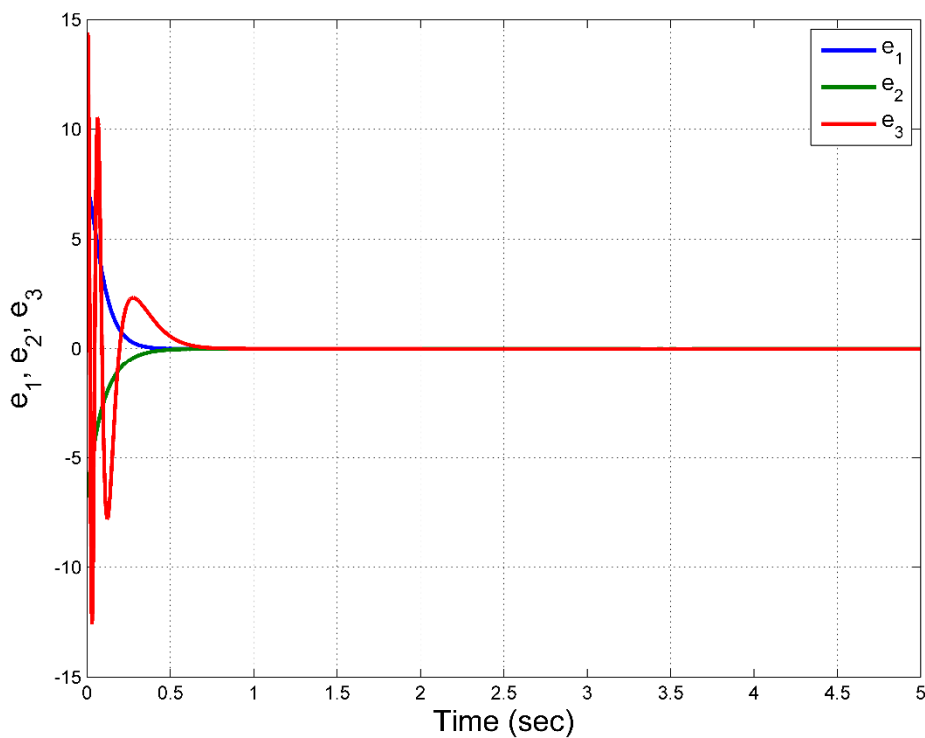
We take the initial conditions of the parameter estimates as

$$\hat{a}(0) = 7.3, \quad \hat{b}(0) = 10.6 \quad (25)$$

Figure 3 shows the synchronization of the states of the new chaotic systems (8) and (9). Figure 4 shows the time-history of the synchronization errors  $e_1, e_2, e_3$ .



**Figure 3.** Complete synchronization of the new chaotic systems



**Figure 4.** Time-history of the synchronization errors for the new chaotic systems

#### 4. Circuit implementation of the new chaotic system

The electronic circuit modeling the new chaotic system (1) is realized by using off-the-shelf components such as resistors, capacitors, operational amplifiers, and multipliers. The circuit electronic of a new chaotic system (1) by MultiSIM is shown in **Figure 5**. In this work, a linear scaling is considered as follows:

$$\left. \begin{aligned} \dot{x}_1 &= a(2x_2 - x_1) - x_3 \\ \dot{x}_2 &= 2.5x_1x_3 \\ \dot{x}_3 &= 10 - 5x_1^2 - x_3 \end{aligned} \right\} \quad (26)$$

The circuit equation corresponding to each state of the scaled system (26) using Kirchhoff's laws can be obtained as:

$$\left. \begin{aligned} \dot{x}_1 &= \frac{1}{C_1R_1}x_2 - \frac{1}{C_1R_2}x_1 - \frac{1}{C_1R_3}x_3 \\ \dot{x}_2 &= \frac{1}{C_2R_4}x_1x_3 \\ \dot{x}_3 &= \frac{1}{C_3R_7}V_1 - \frac{1}{C_3R_5}x_1^2 - \frac{1}{C_3R_6}x_3 \end{aligned} \right\} \quad (27)$$

where the variables  $x_1$ ,  $x_2$ , and  $x_3$  are the outcomes of the integrators  $U1A$ ,  $U3A$ ,  $U5A$ .

We choose  $R_2 = R_8 = R_9 = R_{10} = R_{11} = 100 \text{ k}\Omega$ ,  $R_1 = 50 \text{ K}\Omega$ ,  $R_3 = R_6 = 300 \text{ K}\Omega$ ,  $R_4 = 120 \text{ K}\Omega$ ,  $R_5 = 60 \text{ K}\Omega$ ,  $R_7 = 30 \text{ K}\Omega$ ,  $C_1 = C_2 = C_3 = 3.2 \text{ nF}$ . The supplies of all active devices are  $\pm 15 \text{ Volt}$ . The designed electronic circuit is implemented in MultiSIM. The obtained results are presented in **Figures 6** (a) - (c), which show the phase portraits of the chaotic attractor in  $x_1$ - $x_2$ ,  $x_2$ - $x_3$  and  $x_1$ - $x_3$  planes, respectively. It is easy to see a good agreement between the circuital attractor and theoretical attractor.

#### 5. Conclusions

This work announced a new chaotic system with two quadratic nonlinearities. First, the qualitative properties of the new chaotic system are reported. Dynamical behaviors of the new chaotic system with two quadratic nonlinearities are investigated through equilibrium points, projections of chaotic attractors, Lyapunov exponents, and Kaplan–Yorke dimension. In addition, adaptive synchronization scheme of new chaotic systems is shown via adaptive control approach. Furthermore, an electronic circuit realization of the new chaotic system using the electronic simulation package MultiSIM confirmed the feasibility of the theoretical model.



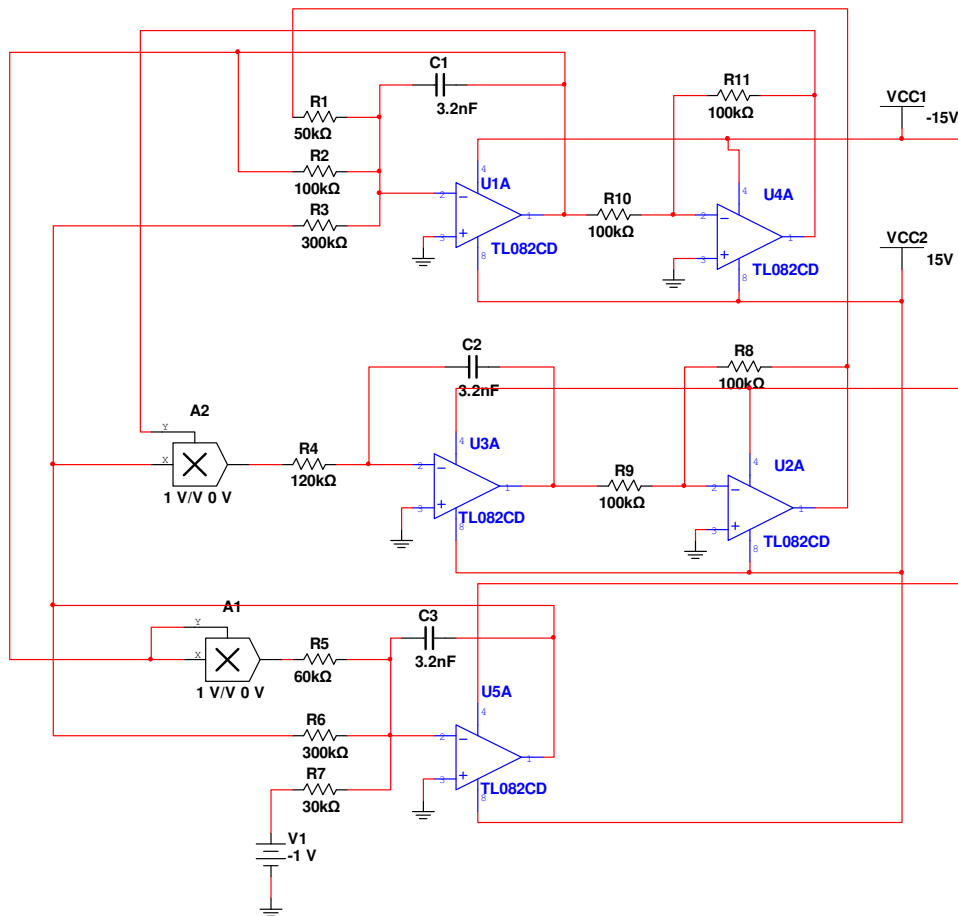
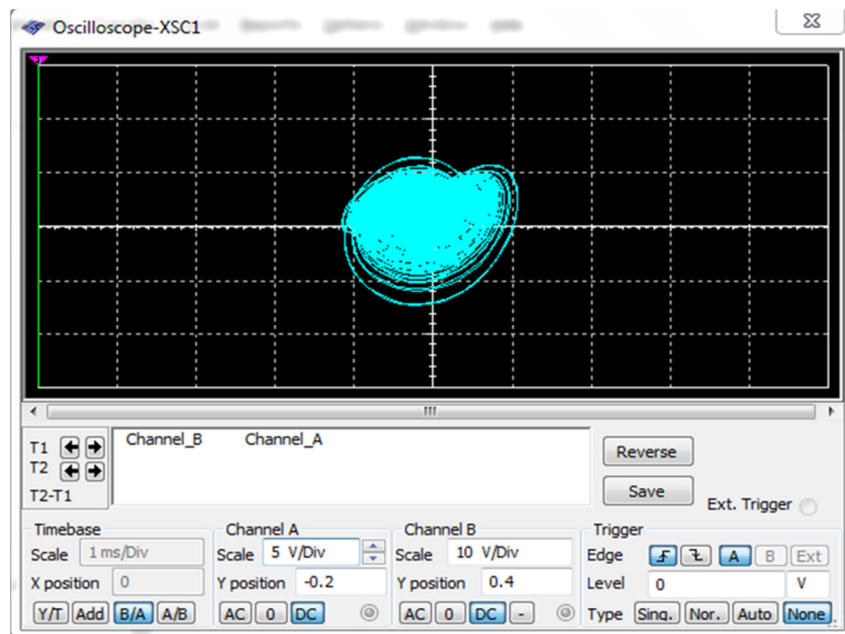
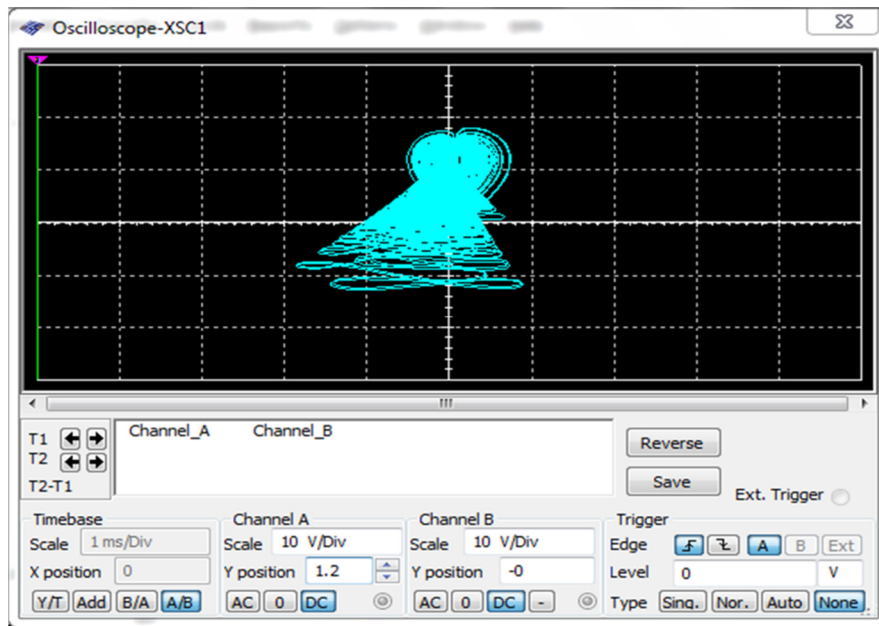


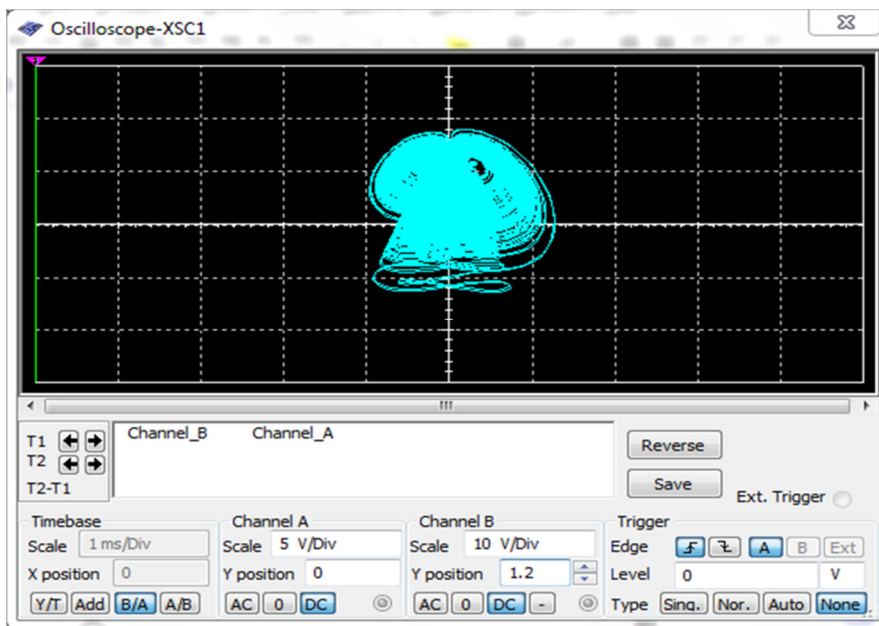
Figure 5 Circuit design for new chaotic system (1) by MultiSIM



(a)



(b)



(c)

**Figure 6** The phase portraits of new chaotic system (1) observed on the oscilloscope in different planes (a)  $x$ - $y$  plane, (b)  $y$ - $z$  plane and (c)  $x$ - $z$  plane by MultiSIM

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